Modeling geocomplexity: "A new kind of science"

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ABSTRACT

A major objective of science is to provide a fundamental understanding of natural phenomena. In "the old kind of science" this was done primarily by using partial differential equations. Boundary and initial value conditions were specified and solutions were obtained either analytically or numerically. However, many phenomena in geology are complex and statistical in nature and thus require alternative approaches. But the observed statistical distributions often are not Gaussian (normal) or log-normal, instead they are power-laws. A power-law (fractal) distribution is a direct consequence of scale invariance, but it is now recognized to also be associated with self-organized complexity. Relatively simple cellular-automata (CA) models provide explanations for a range of complex geological observations. The "sand-pile" model of Bak — the context for "self-organized criticality" (SOC) — has been applied to landslides and turbidite deposits. The "forest-fire" model provides an explanation for the frequency-magnitude statistics of actual forest and wild fires. The slider-block model reproduces the Guttenberg-Richter frequency-magnitude scaling for earthquakes. Many of the patterns generated by the CA approach can be recognized in geological contexts. The use of CA models to provide an understanding of a wide range of natural phenomena has been popularized in Stephen Wolfram's best selling book A New Kind of Science (2002). Since CA models are basically computer games, they are accepted enthusiastically by many students who find other approaches to the quantification of geological problems both difficult and boring.

Keywords: cellular automata, chaos, complexity, computational equivalence, drainage networks, earthquakes, forest fires, fractals, geocomplexity, landslides, seismicity, self-organized complexity, topography.

INTRODUCTION

The geosciences cover a very broad range of topics. In this paper I will concentrate primarily on geological problems. These problems also cover a broad range of topics, i.e. the morphology of the earth's surface, the distribution and origin of rock types, the processes of sedimentation, the distribution of earthquakes in space and time. Massive amounts of data have been collected and stored in repositories. A fundamental question is whether this data can be understood in terms of models and other approaches such as statistical studies.

Geostatistics has played an important role in quantifying a wide range of geological processes. The Gaussian (normal) and Poisson distributions are applicable to truly random processes. Log-normal distributions have also been applied in geology; for example, to the distributions of particle sizes, thicknesses of stratigraphic units, and to mineral and petroleum reserves. A wide variety of other statistical distributions have also been used on a strictly empirical basis.

Analytic and numerical solutions to partial differential equations have been used to explain many observations in physics. In many ways physics is a much simpler (less complex) subject than geology. The motions of particles satisfy relatively simple equations such as Newton's equations of motion. These equations are models and no one could imagine any introductory physics course without these equations. In the 19th century a variety of models were used to explain the behavior of the solid earth. The wave equation explained the propagation of seismic waves. LaPlace's equation explained variations in the surface gravity.

Our understanding of fundamental geological processes was revolutionized by the introduction of the plate tectonics model. This is a simple kinematic model that has a wide range of implications. Today, every introductory textbook in geology incorporates the fundamentals of plate tectonics.

Since the advent of plate tectonics in the late 1960's, many advances have been made in modeling and quantifying geological processes and phenomena. A focus of these studies has been "geodynamics" (Turcotte and Schubert, 2002). A variety of problems have been attacked and solved using classical physics. One example is the half-space cooling model applied to the bathymetry, heat-flow, and gravity anomalies associated with mid-ocean ridges. The hot rock introduced by volcanism at the ridge crest is cooled by conductive heat transport to the sea floor. The cooler rock is more dense and sinks to provide the morphology of ocean ridges. It would clearly be desirable for all undergraduate geology students to learn the fundamentals of elasticity, fluid-mechanics, and heat transfer. Some of this is done in every core course in structural geology and/or tectonics. However, many students who are highly motivated geologists abhor the rigors of partial differential equations, tensor analysis, etc.

It is also true that the deterministic models associated with classical physics cannot address many fundamental problems in geology, as many of these problems have a statistical component. Examples include landforms, river networks, the distributions of mineral deposits and petroleum reservoirs, and the distributions of earthquakes. Problems in this class are said to exhibit "self-organized" complexity. The classic example of self-organized complexity is fluid turbulence. Turbulence is statistical but it is not strictly random. Turbulence is an example of deterministic chaos. The governing equations yield deterministic solutions but these solutions are unstable to small deviations so that the results are not predictable. A consequence is that the weather can be forecast only on a statistical basis.

A new approach to a variety of geological problems has evolved from the fundamental concepts of self-organized complexity. Many of these concepts were developed in the geological sciences in general, and in geology in particular. The first is the concept of fractals developed by Mandelbrot (1967) to explain the length of a rocky coastline. It became clear immediately that other examples of fractality included the Horton-Strahler classification of drainage networks and the Guttenberg-Richter frequency-magnitude relation for earthquakes.

The second fundamental concept in self-organized complexity is deterministic chaos. Lorenz (1963) discovered deterministic chaos while studying a simple model for thermal convection in a fluid, the Lorenz equations. Solutions to these equations were extremely sensitive to initial conditions. Thus the behavior of the equations were not "predictable". Only "statistical forecasts" could be made. It became recognized that fluid turbulence is a direct consequence of deterministic chaos. The equations that govern the behavior of a fluid are well known, the Navier-Stoker equations. But fluid turbulence is a statistical phenomena consisting of fluid swirls and eddies over a wide range of scales.

A second simple model that exhibits deterministic chaos is a pair of slider blocks. If a single slider block is pulled along a surface with a spring the frictional interaction with the surface will result is periodic slip events. These events are predictable and resemble regularly spaced earthquakes on a fault. But if two slider blocks connected with a spring are pulled along a surface with two other springs, the result can be chaotic. Although the governing equations can be specified, the frictional interactions with the surface result in deterministic chaos. The behavior of the system is not predictable. The conclusion is that tectonics and seismicity are also chaotic and therefore are not predictable in a deterministic sense. Only statistical forecasts for the occurrence of earthquakes are possible as is the case for weather.

Although the concept of deterministic chaos was introduced in 1963 prior to the introduction of the fractal concept in 1967, the applicability of deterministic chaos to natural complex phenomena was widely questioned. The original studies of chaos were limited to "low order" systems, the behavior of the systems was complex, but the systems themselves were not.

The third fundamental concept in self-organized complexity is self-organized criticality (SOC). *Bak et al.* (1988) introduced the concept in order to explain the behavior of a simple cellular automata (CA) model. In this model a square array of boxes is considered. Particles are dropped into randomly selected boxes until a box has four particles, these particles are moved to the four adjacent boxes or are lost from the array in the case of edge blocks. Multiple redistributions often result and are referred to as "avalanches". The frequency-area distribution of avalanches is fractal. This is referred to as a cellular-automata (CA) model because it consists of an array of cells (boxes) and only nearest neighbor cells (boxes) interact with a well defined "automata" rule, the redistribution rule.

Another simple CA model that exhibits SOC behavior is the multiple slider block model (*Carlson and Langer*, 1989). Instead of the two connected slider blocks discussed above, a large array of connected slider blocks is considered. It was found that the frequency-area distribution of slip events was fractal. The progression from a pair of slider blocks that exhibited low-order deterministic chaos to a large array of slider blocks that exhibited SOC and fractal behavior related the concepts of chaos and fractality to complex systems, i.e. self-organized complexity.

Models that exhibit self-organized complexity utilize computer based simulations. Two examples were given above and will be considered in more detail in a later section. I will argue that the introduction of some of these simulations into undergraduate geology courses is not only possible but is also very desirable. They are basically computer games and can be very illustrative of fundamental processes in geology. Simulations that exhibit self-organized critical behavior are examples of the use of cellular automata (CA). Stephen Wolfram's recent book, A New Kind of Science (Wolfram, 2002), argues that CA simulations, broadly interpreted, will lead to a new scientific revolution. Topics addressed include general relativity, computability, turbulence, genetic coding, and many others. This book is a best seller, but is also extremely controversial. There is no absolute definition of a CA. It must be a discretized system, i.e., a one-, two-, three-, or higher dimensional grid of points. In many applications, a square grid of points is considered. Usually, but not always, interactions are only with nearest neighbors.

FRACTALITY

The concept of fractals was introduced by Mandelbrot (1967) to quantify the length of a rocky coastline. Fractality implies the applicability of a fractal distribution

$$N \sim r^{-D} \tag{1}$$

where N is the number of objects with a linear dimension greater than r and D is the fractal dimension (*Mandelbrot* 1982, *Feder* 1988). Thus, as the size of the representative object decreases, a power-law increase in the number of objects of that size is found. Many distributions in geology satisfy this relation, examples include fragmented rocks, planetary craters, faults and joints, earthquakes, and drainage networks (*Korvin* 1992, *Turcotte* 1997). The applicability of a fractal distribution implies "scale invariance"; it is the only statistical distribution that is scale invariant. In other words, no characteristic length scale is needed to constrain a power-law (fractal) distribution; it applies regardless of the base unit in which r is measured. The near-universal applicability of scale invariance in geology explains the requirement that an object with a well defined scale must be included in most geological photographs. Courses in fractality are given at many high schools, and they are often popular even with students who dislike mathematical subjects such as trigonometry and geometry. A simple example illustrating fractality in an introductory geology laboratory is to measure the length of a closed contour on a topographic map using a fixed-length divider. In general, a plot of the number of divider steps versus the divider length satisfies Eq. (1).

DETERMINISTIC CHAOS

The concepts associated with deterministic chaos are very popular with students; these are set forth by Gleick (1987), Mullin (1993), and Peak and Frame (1994). Lorenz (1963) discovered deterministic chaos while studying a set of three total differential equations applicable to thermal convection. Solutions to these equations exhibit extreme sensitivity to initial conditions — their temporal evolution is not predictable. Subsequently, many other sets of nonlinear differential equations have been found to exhibit this behavior. Mantle convection and convection in the earth's core are certainly examples of chaotic convection.

From a student's point of view, the best illustration of deterministic chaos comes from the study of simple recursive formulas (maps). May (1976) showed that the logistic map illustrates all aspects of deterministic chaos very simply. The logistic map is defined by the recursion relation

$$x_{n+1} = ax_n(1 - x_n) \tag{2}$$

First, the value of the constant a must be specified. Then an initial value of x_n, designated x_0, is specified. Substitution of $x_n = x_0$ into Eq. (2) gives x_1, the next substitution of $x_n = x_1$ into Eq. (2) gives x_2, and so forth. The extreme sensitivity of solutions to initial conditions is illustrated by the following example. Take $a = 4$ and $x_0 = 0.8$, and, using Eq. (2), we find $x_1 = 0.64$. Using this value for x_n in Eq. (2), we find $x_2 = 0.9216$. After three further iterations, we find $x_5 = 0.585\ldots$ Now repeat the process taking $a = 4$ and $x_0 = 0.81$. After five iterations, we find $x_5 = 0.915\ldots$ The initial values (0.81 and 0.80) differ by 1.25%, while the values after five iterations (0.585 and 0.915) differ by 56%. This is an illustration of the extreme sensitivity to initial conditions exhibited by deterministic chaos. Weather and seismicity are natural examples of deterministic chaos. Neither is predictable in a deterministic sense — only probabilistic forecasts can be made.

SELF-ORGANIZED CRITICALITY

The concept of self-organized criticality (SOC) (*Bak* 1996, *Jensen* 1998, *Turcotte* 1999) was introduced by Bak et al. (1988) to explain the behavior of a simple CA model. In this model, a square array of boxes is considered. A simple example with four boxes (numbered 1-4) is shown in Figure 1. Particles are randomly dropped into the boxes. In our initial state, Figure 1(A), we have two particles in box 1, three particles in box 2, one particle in box 3, and three particles in box 4. If a particle is dropped into a box with two or fewer particles, it remains in the box. However, if the randomly selected box already contains three particles, it becomes unstable, and the four particles are evenly redistributed to the four neighboring boxes. In this example, box 4 is randomly selected for the addition of a particle as shown in Figure 1(B). Since the box already holds three other particles, a redistribution of the four particles is required as shown in Figure 1(C). The resulting redistribution is illustrated in Figure 1(D). In this state, there are now two particles in box 1, four particles in box 2, two particles in box 3, and zero particles in box 4. Two particles have been lost from the grid. Since there are now four particles in box 2, a second redistribution of particles is required as shown in Figure 1(E). This redistribution of particles is illustrated in Figure 1(F). There are now three particles in box 1, zero particles in box 2, two particles in box 3, and one particle in box 4. Another two particles have been lost from the grid.

Multiple, or chain-reaction, redistributions are referred to as avalanches. Simulations with large numbers of boxes have shown that the number-area distribution of avalanches is fractal and satisfies Eq. (1). The area A of an avalanche, which is the number of boxes that participate in the multiple redistributions, determines the value of r in Eq. (1) via the relation . For this model, it is found that . Since only nearest-neighbor boxes are involved in each redistribution, this is a CA model. This model is referred to as a sandpile model because of the relationship to the behavior of a pile of sand on a square table when individual grains of sand are added to the pile.

A simple demonstration of this behavior uses a cylindrical glass or plastic tube. The tube is half filled with sand, glass beads, or rice and is rolled on a surface. When the flat sand surface reaches the “angle of repose”, about 30˚, avalanches of various sizes occur to maintain the angle. A number of published studies have shown that in some cases a fractal distribution of laboratory avalanches is found (*Nagel* 1992).

Naturally occurring avalanches are also found to satisfy fractal frequency-area statistics. It has been demonstrated convincingly that the frequency-area distribution of large landslides is fractal and satisfies Eq. (1) (*Malamud et al.*, 2004). The metastable area over which a landslide spreads once triggered is analogous to the metastable region over which an avalanche spreads in the sand-pile model due to multiple redistributions.

The behavior of the sandpile model has also been associated with the fractal distribution of turbidite deposits (*Rothman et al.* 1994).

A second CA model that exhibits SOC is the forest-fire model (Drossel and Schwabl, 1992). In this model, a square grid of sites is considered. Each site may or may not have a tree on it (like a partially planted orchard). A simple example with 16 sites (numbered 1-16) is shown in Figure 2. Trees and matches are randomly dropped on sites. If a tree is dropped on an unoccupied (treeless) site, it is planted there. If a match is dropped on an occupied site, the tree at that site "burns." Any tree at a site adjacent to a burning tree also burns. In this way, a model forest fire spreads from site to site until no burning tree is adjacent to a non-burning tree. After the forest fire has spread to its maximum extent, any site that holds a burning tree becomes unoccupied (The affected trees burn completely away).

In our initial state, Figure 2(A), trees are planted on sites 1, 2, 8, 9, 10 and 13. At the next step, site 15 is randomly selected, and a tree is planted at that site as shown in Figure 2(B). In subsequent steps, trees are randomly planted at site 4, as illustrated in Figure 2(C), and at site 5, as illustrated in Figure 2(D). The tree planted at site 5 joins a cluster of two trees and a cluster of three trees into a single cluster of six adjacent trees. At the next step, shown in Figure 2(E), site 10 is randomly selected and receives a dropped match. The tree on this site ignites, and the model fire spreads through the entire six-tree cluster (sites 1, 2, 5, 9, 10 and 13), clearing those sites. The state of the model after the fire has occurred is illustrated in Figure 2(F).

Simulations with large numbers of sites have shown that the number-area distribution of model fires is fractal and satisfies Eq. (1). The area A of a model fire, which is defined to be the number of sites on which the model trees burn, determines the value of r in Eq. (1) via the relation . Just as in the case of the sand-pile model, it is found that . Again, this is a CA model, since fires spread to nearest-neighbor trees. It has been shown by Malamud et al. (1998) that the frequency-area distribution of actual forest fires is fractal and satisfies Eq. (1).

A third CA model that exhibits SOC is the slider-block model (Carlson and Langer, 1989). This model is illustrated in Figure 3. A square array of blocks, each with mass m, is pulled across a surface by a driver plate moving at a constant speed v. The blocks are connected to the driver plate by leaf springs that have spring constant k_p. Each block is connected to its neighboring blocks by leaf or coil springs that have spring constant k_c. The blocks interact with the surface through friction, and the simplest friction law is the static-dynamic law: If the block is stationary (static), the frictional force has a maximum value F_S. If the block is slipping, the frictional force has the dynamic value F_D.

If the static value F_S is greater than the dynamic value F_D, the blocks exhibit "stick-slip behavior": A block is stationary until the force exerted by the puller spring increases the net force on the block to a value equal to F_S. At this point, the block begins to slip, and, as a result, transfers force to the adjacent blocks through the connector springs. These forces may cause one or more of the adjacent blocks to slip as well. The slip event can grow in much the same way that an avalanche grows in the sand-pile model or a forest fire spreads in the forest-fire model.

Once again, simulations with large numbers of blocks have shown that the number-area distribution of slip events is fractal and satisfies Eq. (1). The area *A* of a slip event, which is the number of blocks that slip during the event, once again determines *r* via . Also as before, it is found that .

Slider-block models are simple analogs for repetitive earthquakes on faults (Burridge and Knopoff, 1967). Under almost all circumstances, the number *N* of earthquakes with magnitudes greater than *M* that occur in a region satisfies the Guttenberg-Richter relation

(3)

where *a* and *b* are constants. However, the earthquake magnitude *M* is related to the rupture area *A* by

(4)

where c is another constant. Combining Eqs. (3) and (4) gives

(5)

This relation is entirely equivalent to Eq. (1) with a fractal dimension .
Thus, earthquakes also obey a universal fractal frequency-area distribution.

Frequency-area distributions have important practical applications. For example, the Guttenberg-Richter relation given in Eq. (3) is used to estimate the seismic hazard in a region. In southern California, which is seismically quite active, there are on average about ten M = 4 earthquakes per year. Since it is observed that for active regions, Guttenberg-Richter extrapolates that southern California will experience one M = 5 earthquake each year and one M = 6 earthquake every ten years.

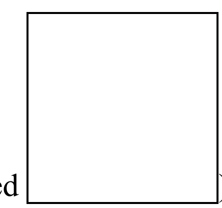
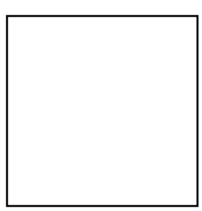
A NEW KIND OF SCIENCE

Geological problems have played an essential role in the development of a number of aspects of complexity. The concept of SOC evolved from a number of simple CA models, three of which were the sand-pile, forest-fire and slider-block models considered in the previous section. These three models behave in remarkably similar ways. While the sand-pile and forest-fire models are stochastic (i.e., involve random selection processes), the slider-block model is completely deterministic (i.e., non-random).

In A New Kind of Science, Wolfram (2002) introduced the principle of computational equivalence. This principle implies that some fundamental problems share a basic equivalence that leads to universal behavior patterns such as those exhibited by SOC. To explore this principle, Wolfram illustrates the complexities that can be generated using a simple one-dimensional (one-row) CA model. An example of such a CA model over a progression of time steps is illustrated in Figure 4(A). At every time step, each cell is either black or white. The color of each cell at the next time step is determined by a set of rules based on the cell's current color and the current colors of its two neighboring cells. In our example, we have a fifteen-cell row, and only cell x_8 is black at initial time step t_1. This model's eight required rules, illustrated in Figure 4(B), specify the future cell color generated by each of the eight possible permutations of black and white cells.

These rules are best understood by a specific example. Consider time step t_2 for cell x_8 in Figure 4(A). At time step t_1, cell x_8 was black, and both of its neighbors, cells x_7 and x_9, were white. Looking at the rule set in Figure 4(B), we see that the pattern white-black-white corresponds to Rule 3. We also see that Rule 3 turns the center cell white during the next time step. Thus, in Figure 4(A), cell x_8 turns white at t_2. Similarly, at time step t_1 Rule 4 applies to cell x_7 (and its neighbors), so cell x_7 is black at t_2. At time step $t_{1,}$ Rule 2 applies to cell x_9, so cell x_9 is also black at t_2. Rule 1 is applicable to the remaining cells at time cell t_1, so all of them are white. Note that cells at the left and right edges are assumed to border white cells. The numbers of the applicable rules for time steps t_2 through t_8 are given for each cell in Figure 4(A).

A distinct, two-dimensional pattern of black squares is formed when successive iterations of this cellular automaton are pictured 'stacked' on top of one another as they are in Figure 4(A). This pattern is a well-known fractal called a Sierpinski gasket. The Sierpinski gasket consists of triangular patterns of different sizes (called orders). The three black squares in time steps t_1 and t_2 are arranged in the fundamental (first-order) pattern. In time steps t_1 through t_4, three instances of the first-order pattern are arranged (triangularly) to form the second-order pattern (In a sense, each black square in the first-order pattern is 'replaced' by an entire first-order pattern). The second-order pattern, plainly, consists of nine black squares. Similarly, three instances of the second-order pattern are arranged to form a third-order pattern in time steps t_1 through t_8. The third-order pattern is an arrangement of 27 black squares, or nine first-order patterns.

We now use Eq. (1) to calculate the fractal dimension of the Sierpinski gasket. For a pattern of a particular order n, the linear dimension r_n is defined to be the number of time steps spanned by the pattern (this span is labeled ). N_n is the total number of times the nth-order pattern appears in the entire figure. For the first-order pattern, we have $r_1 = 2$ (because ) and $N_1 = 9$ (the 3-square first-order pattern appears 9 times). For the second-order pattern, we have $r_2 = 4$ ( ) and $N_2 = 3$. For the third-order pattern, we have $r_3 = 8$ ( ) and $N_3 = 1$. (We may also define a single black square to be a 'zeroth'-order pattern with $r_0 = 1$ and $N_0 = 27$.) By manipulating Eq. (1), we can use the N_n and r_n values for two different pattern orders (i.e., two different values of n) to solve for the fractal dimension D of the Sierpinski gasket. We arbitrarily choose the first- and second-order patterns (i.e., $n = 1$ and $n = 2$) to get

(6)

With the values given above, Eq. (6) yields for the Sierpinski gasket.

This fractal construction can be extended for very large rows of cells over very large numbers of time steps, and the fractal pattern will remain. Figure 4(C) shows the iteration of a row of 1023 cells over 512 time steps. In the initial time step, only cell 512 is black, and subsequent cell colors are chosen using the rules in Figure 4(B). The

Sierpinski gasket is a fantastic visual aid for the classroom because its clear depiction of scaling allows students to literally see fractal structure.

It should be noted that there are 256 distinct rule sets of the form laid out in Figure 4(B). They differ in whether each rule generates a white or black cell in the next time step. The complexity of the applications of these sets is given in Ch. 3 of Wolfram (2002) where the the rule set illustrated in our Figure 4 is labeled 'Rule 90.' Wolfram (2002) goes on to relate the concepts of CA models to computational universality and a wide range of problems in mathematics, the physical, biological and social sciences, philosophy, art and technology. However, we will now return to a couple of further applications of CA models to geological problems.

TOPOGRAPHY AND DRAINAGE NETWORKS

Topography and the associated drainage networks are examples of self-organized complexity in geology. We will first consider drainage networks (*Rodriguez-Iturbe and Rinaldo*, 1997). The fractal nature of drainage networks was recognized by Horton (1945) and Strahler (1957), long before the concept of fractals was introduced by Mandelbrot (1967). A typical drainage network is given in Figure 5A and the stream ordering system is illustrated in Figure 5B. A stream with no upstream tributaries is a first-order stream. When two first-order streams combine, they form a second-order stream. When two second-order streams combine, they form a third-order stream, and so forth. When streams of unequal orders combine, the stream remains a stream of the higher of the two orders.

Horton (1945) introduced two scaling relations for river networks. The first is the bifurcation ratio R_b, defined for a particular drainage basin as the ratio of the number N_n of streams of order n to the number N_{n+1} of streams of order n+1,

(7)

The second relation is the length-order ratio R_r, defined for a particular drainage basin as the ratio of the mean length r_{n+1} of streams of order n+1 to the mean length r_n of streams of order n,

(8)

For a given drainage basin, R_b and R_r are found to be near constant over a range of stream orders. From Eqs. (6), (7) and (8), the fractal dimension D of the drainage basin is defined by the relation

(9)

Drainage basins typically satisfy fractal scaling with a fractal dimension (*Pelletier* 1999). A student can easily determine the orders of the streams in Figure 5A. Using the numbers of streams of each order N_i, N_1, N_2, $\cdots$, N_5, four values of the bifurcation ratio R_b can be obtained from Eq. (7). It will be seen that these values are nearly equal. Other drainage networks, on which to carry out this exercise, can be obtained by tracing them from topo maps.

A simple CA model that generates self-similar networks is the diffusion-limited aggregation (DLA) model (Witten and Sander, 1981). In this model, a square grid of sites is considered. A seed particle is introduced at the center of the grid and it remains in place. Moving particles are added at randomly selected sites on the boundaries of the square grid. These particles follow a random walk until either (1) the particle goes off the grid and is lost or (2) moves to a site adjacent to the growing cluster and sticks to the cluster. In a random walk one of the four adjacent sites is selected randomly and the particle is moved to that site. A typical DLA network (cluster) is illustrated in Figure 6A. The bifurcation ratios and length-order ratios defined in Eqs. (7) and (8) are also found to be constant to a good approximation. Thus DLA networks are fractal and using Eq. (9) it has been found that $D \approx 1.56$, a value somewhat less than those of drainage networks. DLA models have been applied to understand a variety of dendritic growth patterns in igneous rocks and other minerals (Fowler, 1990).

A modification of the DLA model described above does produce networks that are statistically identical to drainage networks (Masek and Turcotte, 1993). Again, a square grid of sites is considered. A number of seed particles are placed along one or more boundaries of a square region. Additional particles are added to randomly selected unoccupied sites in the interior of the grid. The particles are allowed to randomly "walk" through the grid until they reach a site adjacent to the growing network. An example of this procedure is illustrated in Figure 6(B). The growth of the network is closely analogous to the headward migration of an evolving drainage network. Such a network is fractal, typically with .

We next examine topography along a linear, or one-dimensional, track. Let us say you are in a mountainous area. You start at a specified starting point and walk north in a straight line and record the changes in elevation relative to the starting point at equally spaced intervals, say every hundred meters. You then return to the starting point and repeat the procedure in the east, south, west directions. The question is whether the changes in elevation are predictable, at least in a statistical sense. Of course the same data could be collected using a topo map with much less effort.

In order to answer the question it is necessary to introduce the concepts of a Gaussian "white noise" and a Brownian walk. Consider equally spaced points along the x-axis $x_1, x_2, x_3, \cdots x_n$. At each point a y value, $y_1, y_2, y_3, \cdots y_n$ is selected randomly from a Gaussian distribution of values. The sequence of y values is a Gaussian white noise.

Adjacent points in the sequence are uncorrelated. The mean

(10)

and standard deviation δ

(11)

of the sequence are equal to the values for the initial Gaussian distribution.

A Brownian walk is the running sum of a Gaussian white noise. From the sequence of values $y_1, y_2, y_3, \cdots y_n$, we write $z_1 = y_1$, $z_2 = z_1 + y_2$, $z_3 = z_2 + y_3$, $\cdots$. A plot of these z-values against x is a Brownian walk. A Brownian walk exhibits a drift. Starting out at the same point a number of Brownian walks are carried out. The standard deviation of the values δ at a distance L from the starting point is determined using (11) and it is found that

(12)

The standard deviation of the walk increases as the square root of the length of the path from the origin.

The changes in elevation as a function of the distance from a starting point as discussed above is well approximated by a Brownian walk (Ahnert, 1984). Students can easily demonstrate this by obtaining a histogram of the changes in elevation along a series of linear tracks of length L from a common starting point in a mountainous region from a topographic map. The standard deviation of the histogram is obtained using Eq. (11) and the results will be well approximated by Eq. (12). The derived constant C is a measure of the roughness of the topography.

A simple CA model for deposition also gives this behavior (Pelletier and Turcotte, 1997). Consider a linear set of sites on which "particles" are randomly dropped. If the randomly selected site on which a particle falls is lower (has fewer "particles") than its adjacent sites, the particle remains there. If either adjacent site is lower, the particle is moved to that site. The rules for this model are illustrated in Figure 7(A), while the results of a simulation using this model are given in Figure 7(B). In the latter figure, the numbers of particles (heights) are given for the 1024 sites on a linear track. The end result is entirely equivalent to elevations along a linear track. The sequence of the elevations (number of particles on each site) is a Brownian walk — the average difference in elevation Δh between two points is related to the distance L between the two points by Eq. (12). Again, this is a very simple model that students can analyze. This analysis is also applicable to the temporal variability of sandy coastlines. The seaward migration and recession of a sandy coastline is well approximated by a Brownian walk (Tebbens et al., 2002).

DISCUSSION

A major objective of any geology curriculum must be to provide the students with the best understanding of geological phenomena possible under the circumstances. The major point of this paper is that the fundamental aspects of complexity are accessible to a wide range of students and that the fundamentals of complexity do provide a basic understanding of many geological observations.

The wide accessibility of high-speed desk-top and lap-top computers allows any student to carry out quite sophisticated numerical simulations. The class of numerical simulations referred to as cellular automata are basically computer games. The results obtained playing these games can be applicable to a wide range of geological phenomena.

Studies of complexity are "A New Kind of Science". Details of the use of cellular automata simulations to understand complexity in a variety of fields has been given by Wolfram (2002).

The first step towards the new science was the appreciation of the importance of fractal distributions. The concept of fractals evolved in a geological context — the length of a rocky coastline (Mandelbrot, 1967). The continuous fractal distribution given by Eq. (1) is not a true statistical distribution because its integral diverges to infinity. For this reason, fractal distributions are not included in courses in statistics. However, fractal distributions are essential to the understanding of many geological phenomena, such as drainage networks, landforms, and earthquakes. Fractal distributions are more widely applicable in geology than standard statistical distributions such as the Gaussian (normal) or log-normal.

This is direct evidence that many geological observations are the consequence of self-organized complex processes such as those illustrated by simple CA simulations. It should also be emphasized that when applying fractal distributions to geological data there are always upper and lower limits to the applicability. In terms of streams and rivers, there is always a shortest stream and longest river.

The discovery of chaos by Lorenz (1963) preceded the introduction of the fractal concept by Mandelbrot (1967). The role of deterministic chaos is certainly important in the "new" kind of science, but only with the discovery of SOC by Bak et al. (1988) did this role become clear. This evolution is clearly shown by the behavior of the slider-block model illustrated in Figure 3. A single slider block is a typical model in the "old" kind of science. A single block pulled by a spring exhibits simple harmonic motion and periodic slip events occur. A pair of slider blocks connected to each other and to a constant velocity driver plate by springs exhibits classic deterministic chaos (Huang and Turcotte, 1990). The future behavior of the blocks is not predictable. Large numbers of connected slider blocks, on the other hand, exhibit SOC. Slip events have a fractal frequency-size distribution that is directly analogous to seismicity in a region. The association of chaos and SOC with seismicity has been interpreted as precluding earthquake prediction (*Geller et al.* 1997).

The basic theme of this paper is that: (1) there are many data sets in geology that exhibit fractal behavior, (2) this behavior cannot in general be obtained from the solutions of the standard partial differential equations, or from standard statistical studies, and (3) that in many cases this behavior can be obtained using simple CA models. This theme is very similar to the theme set forth by Stephen Wolfram in his book, *A New Kind of Science* (2002).

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Figure 1. Illustration of the sand-pile model. Four boxes (numbered 1 to 4) are considered; the initial distribution of particles is given in (A). One of these boxes is selected at random, in this case box 4, and a particle is dropped into this box. As shown in (C), this box is now unstable, and the four particles are redistributed in (D). Two particles are redistributed to the adjacent boxes 2 and 3 and two particles are lost from the grid. Following this redistribution, box 2 is unstable (see (E)) and the four particles in this box must be redistributed as shown in (F). Two particles are redistributed to the adjacent boxes 1 and 4 and two particles are lost from the grid.

Figure 2. Illustration of the forest-fire model. In (A) the numbering of the 16 sites is given and the initial configuration of trees ("occupied" sites) is shown. One of the unoccupied sites is selected at random and a tree is planted on the site. In (B), (C), and (D), new trees are planted on sites 15, 4, and 5, respectively. After a specified number of tree plantings a match is dropped on a randomly selected occupied site. The match starts a fire on that site and the fire spreads to all adjacent occupied sites also burning those trees. A match is dropped on site 10 in (E), and the fire spreads to five adjacent occupied sites. The burned (black) trees are removed from the grid and the configuration after the model fire is given in (F).

Figure 3. Illustration of the slider-block model. Blocks of mass m rest upon the lower plate (the "surface") while attached to the upper plate (the "driver") by leaf springs (the "pullers") with spring constant k_p. Each block is also connected to its nearest neighbor blocks by either coil or leaf springs with spring constant k_c. The linear displacement y_i of

each block (relative to the upper plate) is monitored over time. As the upper plate moves at velocity *v* with respect to the lower plate, the blocks exhibit stick-slip behavior due to the frictional interaction with the lower plate. The slip events have a fractal distribution of sizes similar to the model avalanches associated with the sand-pile model (Figure 1) and the model forest fires associated with the forest-fire model (Figure 2).

Figure 4. (A) A cellular automaton that generates a perfect fractal (the Sierpinski gasket). The evolution of a row of 15 cells over eight time steps is shown. For a given time step, each cell is either black or white. The color of a cell and the colors of its nearest neighbors determine the cell's color in the next time step. This determination is made according to the rule set illustrated in (B). The rule set enumerates the eight possible configurations of colors for a cell, its left neighbor, and its right neighbor during a single

time step . Each configuration specifies either a black or white color for at the next time step . The number inside each cell in (A) indicates which of the eight configurations in (B) determined the cell's color. Note that 'edge' cells are assumed to border white cells. The structure of the black and white cells in (A) is a perfect fractal construction known as a Sierpinski gasket. In (C) the CA construction given in (A) is extended to 1023 cells over 512 time steps. The fractal pattern is sharply defined at this resolution.

Figure 5. (A) A typical drainage network in a mountainous region. The scale is shown. Drainage networks can be obtained directly from topographic maps. (B) Illustration of the Strahler (1957) stream-ordering system. The initial streams are first order, two first-order streams merge to give a second-order stream, and so forth.

Figure 6. (A) A typical diffusion-limited aggregation (DLA) network is illustrated. The growing network started with a single seed particle at the center. The network grew by the addition of particles that followed a random-walk path until they "stuck" to the growing network. (B) Illustration of a synthetic drainage network obtained by a modification of the DLA model. Seven seed particles were placed on the lower boundary of the region. Particles were introduced at randomly selected sites within the region.

These particles followed random-walk paths until they "stuck" to the growing network. The network grew from the lower boundary upward in direct analogy to the headward migration of an actual drainage network.

Figure 7. The behavior of a simple cellular-automaton model for deposition along a one-dimensional track is illustrated. A linear set of sites is considered and a particle (square box) is dropped on a randomly selected site. (A) The CA rules are illustrated. After the particle (highlighted in gray) has dropped on a randomly selected site it either remains on that site or is moved to an adjacent site according to the rules: (I). A particle dropped on a site lower than both neighboring sites stays. (II). A particle dropped on a site higher than at least one neighboring site migrates to the lower neighboring site. (III). Special case of (II) in which both neighboring sites have the same elevation. The particle migrates to a randomly selected neighboring site. (B). Synthetic topography (linear elevation plot) generated using this model. Particles have been dropped on 1024 sites. On average 1022 particles have been dropped on each site. The number of particles is analogous to height (elevation) and the lattice site number is analogous to horizontal distance. The horizontal variability is a Brownian walk.