**The Use of Quantitative Reasoning across the Curriculum: Empirical Evidence from Carleton College**

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**Abstract**
We examine papers written by first- and second-year undergraduates for submission to a college writing portfolio, coding for the potential relevance of quantitative reasoning (QR) to the argument. (Note: We measure the potential relevance to the argument, not whether students realize this potential by using quantitative evidence.) We distinguish papers in which QR is central to the main thrust of the argument ("centrally relevant") from those in which QR would strengthen the argument by providing context, enriching description, or presenting background ("peripherally relevant"). We find extensive potential for QR instruction across the curriculum. In nearly one-third of papers, QR was centrally relevant and in another quarter of papers, QR was peripherally so bringing the total fraction of QR-relevant papers above 50%. Of equal importance, much QR-relevant writing takes place outside the natural sciences. Of papers for which QR is centrally (peripherally) relevant, around 50% (90%) were written outside STEM courses. Moreover, 35% of papers written in arts, literature, and humanities courses were QR relevant. These findings provide supporting evidence to theorists who have argued that quantitative reasoning can and should be taught throughout the curriculum.

**Introduction**

In his call for reform in higher education, *Our Underachieving Colleges*, Derek Bok (2006) argues that students should encounter quantitative reasoning (QR) throughout the curriculum:

> Like learning to write well or speaking a foreign language, numeracy is not something mastered in a single course. The ability to apply quantitative methods to real-world problems requires a facility and an insight and intuition that can be developed only through repeated practice. *Thus quantitative material needs to permeate the curriculum, not only in the sciences but also in the social sciences and, in appropriate cases, in the humanities,* so that students have opportunities to practice their skills and see how useful they can be in understanding a wide range of problems. (p. 134, emphasis added)

In part, the argument for this cross-cutting approach to QR education rests on a general principle articulated in Shulman (1997): "Authentic and enduring learning…can rarely succeed one course at a time." But Steen (2004) argues that inter-disciplinary instruction is particularly important to QR. "If [QR] remains the responsibility solely of mathematics departments—especially if it is caged into a single course such as 'Math for Liberal Arts'—students will continue to see QL as something that happens only in the mathematics classroom."

While these aggressive calls for QR across the curriculum are exciting, it is not immediately clear that they are practical. Numeracy advocates may easily imagine assignments and course modules that teach Steen's (2004) eloquently described "sophisticated reasoning with elementary mathematics" in the context of history, religion, economics English composition, physics, fine arts, and other disciplines. But the reality is that leaders of the QR movement only teach a small fraction of those courses. Unless we can show that QR is relevant in the context of courses already being taught on topics already being explored by the professors already in the classroom, substantial progress toward fulfilling the visions of Bok and Steen will be very slow.

Based on a case study of Carleton College, this paper provides evidence that QR is indeed relevant throughout the academy in courses as they are currently taught. Our conception of QR is the habit of mind to consider both the power and limitations of quantitative evidence in the evaluation, construction, and communication of arguments in public, professional, and personal life. (See Grawe and Rutz 2009, Lutsky 2008, and Rutz and Grawe 2009 for fuller discussions of this conception of QR and its implications for professional development and the curriculum.) This understanding of QR situated in the context of argument led us to look for evidence of the relevance and use of quantitative evidence in papers written by students in the first sophomore year (more or less the general education curriculum).

In the next section we describe the assessment tool and sampling process that generated the data used in the case study.[1] The results, presented in the third section of the paper, are encouraging. Despite predictable differences in the degree to which the college's four divisions—arts & literature, humanities, social sciences, and natural sciences—explicitly require QR in assignments, opportunities to teach numeracy already extend across the entire campus.[2] In particular

- Over 30% of papers addressed topics for which QR was relevant to a central question, issue, or theme (ie QR was "centrally relevant" to the paper).

[1] Grawe et al. (2009) provide a complete description of the assessment rubric and document its reliability across raters.

[2] Our mapping of departments to divisions is as follows:
Arts & Literature: Arabic, art history, studio art, Asian languages, cinema and media studies, Chinese, classics, dance, English, French, Francophone studies, German, Greek, Hebrew, Japanese, Latin, literary and cultural studies, media studies, music, Russian, Spanish, and theater
Humanities: history, philosophy, and religion
Natural Sciences: astronomy, biology, chemistry, computer science, geology, mathematics, and physics
Social Sciences: archeology, economics, linguistics, political science, psychology, and sociology/anthropology
Interdisciplinary: African-American studies, cross-cultural studies, cognitive studies, environment and technology studies, European studies, interdisciplinary studies, Latin-American studies, and women and gender studies

- In another 25% of papers, QR was “peripherally relevant”— the use of numbers would provide useful detail, enrich descriptions, present background, or establish frames of reference.
- Thus, in Carleton courses as currently taught over 50% of written assignments provide an opportunity for students to engage in QR.
- Of papers for which QR was centrally relevant, almost 50% were written outside the natural sciences.
- Of papers for which QR was peripherally relevant, over 90% were written outside the natural sciences and more than half were written in arts & literature or humanities classes.
- In total, 35% of papers written for arts & literature and humanities classes were QR relevant (either centrally or peripherally).

In total, the case study presents on optimistic picture: As currently configured, ample opportunities exist for students to experience and engage in QR across the curriculum.

**Methods and Data**

The data for this study were generated by an application of the QR-in-writing assessment rubric described in Grawe et al. (2009). In particular, we focus on the data generated by teo rubric items.: “Is QR potentially relevant to this paper?” and “Does the assignment explicitly call for the use of QR in the paper?”

The possible responses to the first question are:

- No or incidentally only
- Yes, but peripherally only

- Yes, centrally.

The rubric codebook provides the following guidance to scorers:

> This is a reader's assessment of the *potential* contribution of quantitative information to the paper based on the stated and implied goals of the paper itself; it is *not* an assessment of the specifications of the assignment.
>
> In making this assessment, consider how a reasonable person would consider the relevance of QR to the topic chosen by the student. That is, ask if you would expect QR to play a peripheral or central role in a strong paper on this topic, not if you could somehow squeeze QR into this context.

The category of "peripherally relevant" QR papers is alluded to in Jane Miller's *Chicago Guide to Writing about* Numbers: "Even for works that are not inherently quantitative, one or two numeric facts can help convey the importance or context of your topic…. For both qualitative and quantitative works, communicating numeric concepts is an important part of telling the broader story" (p. 1). We would only add that the peripheral use of quantitative evidence need not be limited to "one or two numeric facts." For example, consider this excerpt from Tim Flannery's 2007 *New York Review of Books* review of two books written on recent scientific discoveries from deep oceanic explorations.[3]

> Only the uppermost part of the oceans—the top two hundred meters—bears any resemblance to the sunlit waters we are familiar with, yet below that zone lies the largest habitat on Earth. Ninety percent of all the ocean's water lies below two hundred meters, and its volume is eleven times greater than that of all of the land above the sea. This great realm is divided into a twilight zone—between two hundred and one thousand meters deep—and a zone of total darkness, which is itself varyingly subdivided. Below six thousand meters lies a region known as the hadal zone (a term coined only in 1959 from the French *Hadès*); in the Marianas Trench off the Philippines it is 11,000 meters deep. Ships plying the waters over the trench glide as far above Earth's surface as do jet aircraft crossing the face of America.

While neither the review itself not the books it surveys deal centrally with quantitative questions, this example demonstrates a richness of description through the use of numerical evidence.

[3] We thank Kristin Partlo for bringing this example to our attention.

When scoring of the second question—whether QR is explicitly called for in the assignment—raters are given three possible answers: Yes, No, or No Assignment Present. In the norming session that precedes actual scoring, raters are reminded not to make inferences. If the paper is a lab report which clearly required QR but no physical assignment is present, the proper response is "No Assignment Present." Similarly, even if it seems that an assignment would be best met by using QR but there is no explicit demand for quantitative evidence, the proper response is "No."

A team of 17 scorers (15 faculty members and 2 staff) scored papers submitted by students in the 2005 and 2010 graduating classes as part of Carleton's required writing portfolio. Collected from students at the completion of the second year, the portfolio includes three to five essays written in at least two of the four college divisions demonstrating competency in five areas: observation, analysis, interpretation, documented sources, and thesis-driven argument. Students are asked to include copies of associated assignments. We excluded portfolios from students who initially received less than a passing mark when assessed by the writing program (approximately 5% of all students) or who chose not to allow their work to be used for research purposes ( roughly 15% of students). From the resulting population, we drew a random 50% sample of portfolios (388 in total). From each of these portfolios, we randomly chose one of the papers submitted to represent the categories of analysis, interpretation, or observation.[4]

While students are asked to turn in assignment handouts for each paper submitted in their portfolios, our sample of papers included assignments in only 65.3% of cases. Given that one-third of papers did not include an assignment, selection bias might contaminate the data if

[4] We focused on these subgroups because the instructions given to students with the writing portfolio explicitly mention data in descriptions of these three categories. However, we have subsequently learned that students often submit papers written for QR-rich first-year seminars under the documented sources category. As such, the results presented here likely understate the prevalence of QR in the curriculum.

students are more or less likely to include assignments which explicitly require QR. While we cannot test this hypothesis directly because we don't have the assignments that were not turned in, we can look for indirect evidence of selection. For instance, if students are less more to turn in handouts for assignments explicitly requiring QR we would likely see relatively fewer natural science papers in the group of submissions including assignments. Fortunately, an analysis of the correlation between propensity to include the assignment with the division of the course for which the papers were written suggests that failure to include the assignment is random. Table 1 reports the divisional distribution of papers scored by whether students did or did not include an assignment. A chi-square goodness of fit test fails to reject the null hypothesis that the two distributions are the same ($p = 0.964$).

[Table 1 goes here]

Table 2 provides summary statistics for our sample. The demographic data nearly match general enrollment data as one would expect given Carleton's relatively modest drop-out rate. The over-representation of papers written in lower division courses reflects the fact that the portfolios are collected at the end of sophomore year and underscores the fact that the work we are examining might be thought of as coming from the "general education curriculum." In particular, we are not seeing many papers in upper division courses that may reflect methods courses required in quantitative majors. Finally, the data show a bias toward work done in the humanities and away from that done in the natural sciences. Whereas only 14% of courses taken by these students were in the humanities, 21% of papers in our sample were written in these courses. By contrast, while 27% of courses taken were in the natural sciences, only 20% of the papers originated in these disciplines. This surely reflects the fact that writing assignments are more prevalent in the former departments than in the latter.

[Table 2 goes here]

**Results**

One way we might explore the relevance of QR across the curriculum is to ask how frequently assignments in each division explicitly require the use of quantitative evidence. Table 3 summarizes the propensity for assignments to call for QR, separated by divisions. The first row reports statistics for all of the papers in our sample. Around one-third of papers explicitly require QR, but predictable disparities exist across disciplines. In particular, fewer than 10% of arts, literature, and humanities assignments require students to engage quantitative evidence. By contrast among natural science papers only 10% of assignments fail to demand arguments involving numbers. In large part the latter statistic reflects the fact that lab reports (a very common natural science paper type in the portfolio) almost invariably involve the collection and analysis of data. Social science assignments fall in the middle with roughly one third calling for QR.

[Table 3 goes here]

In the second and third rows of Table 3 the data are presented by the level of the course for which the paper was written. Upper division courses are not analyzed separately due to the small sample size (n=10). In both lower and middle level courses roughly one-third of papers explicit call for QR. However, it appears that QR might be required more often in middle level courses in the humanities, social sciences, and natural sciences and less often in arts and literature. These changes would be consistent with the increasing importance of disciplinary norms as students progress into middle and upper level courses. If this is true, attempts to introduce QR into arts, literature, and humanities courses should focus on introductory level classes. While the roughly 10 percentage point changes across course level are meaningful in

magnitude they are not statistically significant. As our sample size grows with future assessment we will want to revisit this question.

Viewed through the less of assignments explicitly calling for students to engage with quantitative evidence, the data paint a bleak picture for those attempting to permeate the curriculum with QR. With less than 10% of papers written in traditionally "non-quantitative" disciplines requiring QR it might seem difficult to convince faculty in the arts, literature, and humanities to spend scarce time thinking about how we might improve numeracy among students. What is more, the data on QR-explicit assignments seem to suggest that a program directors have little benefit in reaching out to faculty in these disciplines. Of papers written for such assignments, nearly 60% fall in the natural sciences and another 26% are written in the social sciences. With only 14% of QR-assigned papers written in the arts, literature, and humanities it might seem foolish to invest resources into programming designed to help faculty in these fields to teach numeracy more effectively.

We would argue that these conclusions are incorrect because important opportunities to teach and learn QR are missed by an approach that focuses solely on the explicit demands of assignments. An examination of the paper topics chosen by students shows extensive opportunities for numeracy instruction in all divisions of the college. Table 4 presents the fraction of papers written in each of the disciplines, dividing the data by the relevance of QR to the argument as approached by the student.

[Table 4 goes here]

The first row shows the percent of papers for which QR was centrally relevant—potential uses of numbers address a central question, issue, or theme. Because the distribution of centrally relevant papers across divisions almost matches that of QR-required assignments reported in

Table 3, we wondered whether the category of papers for which QR is centrally relevant is equivalents to the category of papers written to assignments for which QR is explicitly required. Table 5 shows that there is considerable overlap with the central relevance of QR correctly predicted by the assignment category in just under 90% of all cases for which the assignment is available. Still it appears there is an important and substantial group of papers outside this overlapping region: Among centrally relevant papers almost 20% were written to assignments that did not explicitly demand quantitative evidence.

[Table 5 goes here]

The large fraction of natural science papers requiring QR might seem to support a narrow QR curriculum housed in mathematics and natural sciences. But looking at the data in the opposite direction we see that over 50% of papers for which QR is centrally relevant were written for courses outside these departments. The fact that fully one-third of such papers are written in the social sciences suggests that efforts to reform QR education must be broad enough to include social scientists. The remaining data contained in Table 4 indicates that even this is far too narrow an approach.

The second row of Table 4 presents the fraction of papers written in each division for which QR is peripherally relevant—given the way the student has written the paper the use of quantitative evidence could potentially provide useful detail, enrich descriptions, present background, or establish frames of reference. This data presents and entirely different picture of where QR is happening in the curriculum. First, natural science papers almost never fall in the peripherally relevant category because nearly all papers written in these fields take up an issue for which QR is centrally relevant. As a result, 9-in-10 opportunities to engage QR in this way occur outside the natural sciences. Second, roughly one-fifth of papers written in the arts &

literature and one-third of papers written in the humanities approach their topics in such a way that an engagement with quantitative evidence would be helpful to set a context or provide detail.

To understand better how QR can and does inform papers in arts & literature and humanities courses, consider several examples.

Arts & Lit: English 109 and philosophy of poverty

Humanities: Muslims in France

Note lessons also commonly apply in social sciences: psychogenic pain and franchises as PR, time diary analysis and Denmark as model political system as CR

**Discussion**

The results presented above support the curricular models advocated by Bok and Steen. We believe the data point to two important lessons. First, QR is relevant to courses outside the natural sciences as they are currently taught. Readers may not be surprised to learn that students regularly engage QR-relevant topics in the social sciences: 75.5% of papers written in these departments are QR-relevant and 40.8% are centrally so. If numeracy programs have not already engaged faculty in this group, social scientists would seem to represent a great place to begin building broader campus support.

While social science faculty may be ready allies, we would call particular attention to the role of QR in the arts, literature, and humanities. Fully one-third of papers written for classes in these divisions head in a direction for which QR is relevant. Apparently the general education curriculum as presently configured provides ample opportunity for students to engage in numeracy across the campus. Enticing faculty in these traditionally non-quantitative fields to take up the QR banner will likely require active recruitment. In particular, since less than 10% of assignments in these divisions explicitly call for QR and a similarly small fraction of papers

take up topics for which quantitative evidence plays a central role, we suspect that many faculty in these courses will need to be shown how QR enters arguments in their classes. While the results from this case study might be useful information, it may be more effective to provide examples from actual student work and to draw parallels between the rhetorical uses of numbers we wish to encourage and other rhetorical issues faculty are already aware of. For example, urging students to use actual quantitative evidence rather than rely on quasi-numeric phrases like "many," "few," or "some" can be equated with promoting precision in language or requiring evidence for claims. In other words, we may need to "repackage" the QR discipline to make clear that it complements existing teaching goals in disciplines which view themselves as "non-quantitative."

The second lesson we derive from the data is that natural scientists are unlikely to reform QR attitudes and aptitudes on their own. There is simply too much QR going on in other divisions for this strategy to hold much hope. In our sample, only one-third of QR-relevant papers were written for courses in the natural sciences—the same fraction as written for courses in the arts, literature, or humanities. Even among papers for which QR was centrally relevant, almost 50% were written in other divisions. And of papers for which QR is peripherally relevant, more than half were written in the arts, literature, or humanities. If natural scientists attempt to "go it alone," students will receive very inconsistent messages concerning the power of quantitative evidence in arguments. On some assignments, students will hear QR is very important while on others they will learn that inattention to numeracy has no consequence. As a result, some students will leave our campuses without an appreciation for the power of numerical thought. Worse still, many will come to understand that QR is relevant only to those pursuing careers in a narrow set of fields—an outcome which Steen (p. 18 2004) rightly warns against.

Some may argue that the peripheral use of QR is not "real" numeracy. We would argue against this perspective for 3 reasons. First, there is no limit on the sophistication of skills that might be employed in a peripheral way. While many peripheral uses of quantitative evidence may involve citing a simple mean, it is entirely plausible that a student could reference the results of an advanced statistical analysis as a way of providing detail or context. This is a particular instance of a broader argument made by Steen (p. 16 2004): "[QR] functions at many levels, and no matter how much is accomplished … there will be much to pursue...." That QR is peripheral need not mean it is simplistic.

Second, effective peripheral use of QR often addresses questions which others have argued are "foundational" to the discipline like, "What do the numbers show?", "How representative is that?", and "Compared to what?"[5] In fact, it may be more effective to teach students to ask these questions in a peripheral context where the details of more complex analytical tasks associated with central use are absent.

Third, we believe we must view students' QR educations as a journey through many courses rather than a destination arrived at in a single class. The peripheral QR skills that may be taught in many arts, literature, and humanities courses are surely not all of the numeracy capacities we desire our students to obtain. But by developing foundational skills and habits of mind in these courses prepares students for deeper, more sophisticated applications in other courses in which QR methods are the focus.

Finally, we speculate that repeated encounters with peripheral QR applications in arts, literature, and humanities courses might reduce math phobia. XX and XX have argued that the QR discipline in general can serve to knock down the barriers of math fears. It would seem that

[5] These are three of Lutsky's "10 Foundational Quantitative Reasoning Questions" found at http://serc.carleton.edu/quirk/CarletonResources/10questions.html.

for many students facing this struggle repeated encounters with peripheral QR would be especially powerful. We hope future research can confirm this hypothesis to guide and motivated further curricular reform.

**Conclusion**

In *Achieving Quantitative Literacy*, Lynn Steen (2004) argues that, "As faculty now see improved writing as a goal in most courses, so they should see quantitative literacy as one of the explicit educational goals in courses across the curriculum." This goal, echoed by Bok (2006) and others, is only possible if professors teaching "most courses" view quantitative reasoning (QR) as relevant to their classes and their course goals. This paper assesses the degree to which the use of quantitative evidence is relevant to papers submitted to Carleton College's sophomore writing portfolio. This sample of papers written in the general education curriculum suggests that Steen's goals are achievable. Conceived of broadly, QR is relevant to nearly 60% of the papers in our case study. While the rate of relevance is predictably higher in the natural and social sciences (98.6% and 75.5%), fully one-third of papers written for courses in the arts, literature, and humanities were found to be QR-relevant.

**Table 1: Distribution of division and level of course for which papers were written among papers including and failing to include an assignment handout**

| Division | With handout | Without handout | Total |
|---|---|---|---|
| Arts and Literature | 28.7% | 30% | 29.1% |
| Humanities | 20.9% | 21.5% | 21.1% |
| Natural Sciences | 20.5% | 17.7% | 19.5% |
| Social Sciences | 26.2% | 26.2% | 26.2% |
| Interdisciplinary | 3.7% | 4.6% | 4.0% |
| Sample size | 244 | 130 | 374 |

**Table 2: Summary statistics describing students who wrote scored papers and courses for which the papers were written**

| | | | |
|---|---|---|---|
| **Student demographics** | | | |
| *Sex* | | *Ethnicity* | |
| Male | 45.6% | White | 77.6% |
| Female | 54.4% | African American | 4.0% |
| | | Hispanic | 4.0% |
| | | Asian | 9.1% |
| | | No response or other | 5.3% |
| **Course characteristics** | | | |
| *Level* | | *Division* | |
| Lower | 68.6% | Arts and Literature | 29.1% |
| Middle | 28.7% | Humanities | 21.1% |
| Upper | 2.7% | Natural Sciences | 19.5% |
| | | Social Sciences | 26.2% |
| | | Interdisciplinary | 4.0% |
| Sample size | 374 | | |

**Table 3: Percent of assignments explicitly requiring QR, by division for which paper was written**

| Arts & Literature | Humanities | Natural Sciences | Social Sciences | Inter-disciplinary | All |
|---|---|---|---|---|---|
| *All assignments* | | | | | |
| 8.6% | 5.9% | 92.0% | 32.8% | 22.2% | 32.0% |
| (69) | (51) | (50) | (62) | (9) | (241) |
| *Assignments written for lower-level courses* | | | | | |
| 10.3% | 3.2% | 89.2% | 29.7% | 33.3% | 31.4% |
| (58) | (31) | (37) | (37) | (6) | (169) |
| *Assignments written for middle-level course* | | | | | |
| 0.0% | 10.0% | 100% | 40% | 0.0% | 34.7% |
| (11) | (20) | (13) | (25) | (3) | (72) |

*Note: Number of observations in parentheses.*

**Table 4: Percent of student papers for which QR is relevant**

| Arts & Literature | Humanities | Natural Sciences | Social Sciences | Inter-disciplinary | All |
|---|---|---|---|---|---|
| *Centrally relevant* | | | | | |
| 7.3% | 6.3% | 90.4% | 40.8% | 26.7% | 32.9% |
| (8) | (5) | (66) | (40) | (4) | (123) |
| *Peripherally relevant* | | | | | |
| 22.0% | 32.9% | 8.2% | 34.7% | 26.7% | 25.1% |
| (24) | (26) | (6) | (34) | (4) | (94) |
| *Either centrally or peripherally relevant* | | | | | |
| 29.4% | 39.2% | 98.6% | 75.5% | 53.4% | 58% |
| (32) | (31) | (72) | (74) | (8) | (217) |
| *Irrelevant* | | | | | |
| 70.6% | 60.8% | 1.4% | 24.5% | 46.7% | 42.0% |
| (77) | (48) | (1) | (24) | (7) | (157) |
| *All* | | | | | |
| 100.0% | 100.0% | 100% | 100.0% | 100.0% | 100.0% |
| (109) | (79) | (73) | (98) | (15) | (374) |

*Note: Number of observations in parentheses.*

**Table 5: Overlap between papers for which QR is centrally relevant and papers written to assignment that explicitly require QR**

| | Assignment requires QR? | | |
|---|---|---|---|
| Centrally QR relevant? | No | Yes | All |
| No | 89.8%<br>(150) | 11.5%<br>(9) | 64.9%<br>(159) |
| Yes | 10.2%<br>(17) | 88.5%<br>(69) | 35.1%<br>(86) |
| All | 100.0%<br>(167) | 100.0%<br>(78) | 100.0%<br>(245) |

*Note: Number of observations in parentheses.*