Jerry Johnson
University of Nevada
July, 2002

UNDERSTANDING BIG NUMBERS

Suggested grade levels: 9 and up.

Possible subject areas: Social studies, economics, science.

Math skills: Arithmetic. Exponents. Conversion of units of measurement. Scientific notation is helpful for estimation.

Overview: In today's world, large numbers are common. This module is designed to help put them in some perspective by estimation and understanding relative sizes. It also includes some fun questions about how big a billion dollars is.

UNDERSTANDING BIG NUMBERS: Student Activities

A million is a thousand thousand or 1,000 x 1,000 = 1,000,000. A billion is a thousand million 1,000 x 1,000,000 = 1,000,000,000. A trillion is a thousand billion 1,000 x 1,000,000,000 = 1,000,000,000,000. As examples of very big numbers, consider these:

- According to the U.S. Bureau of the Census, the resident population of the United States, projected to 7/5/2002 at 2:57:52 PM EDT is 287,452,022. www.census.gov/

- According to the International Programs Center, U.S. Bureau of the Census, the total population of the World, projected to 7/5/02 at 18:56:23 GMT (7/5/02 at 2:56:23 PM EDT) is 6,235,251,506. *www.census.gov/ipc/www/*

- The 2002 National Budget is $2.052 trillion dollars. *www.whitehouse.gov/omb/budget/fy2003/bud34.html*

- The U.S. National Debt Clock says, "The Outstanding Public Debt as of 03 May 2002 at 09:11:22 PM GMT is: $6,048,946,190,404.13." That's more than 6 trillion dollars. *www.brillig.com/debt_clock/*

Although they are important, numbers this large are hard to comprehend. Sometimes estimation and comparisons are useful.

1. Using the figures above, what is each citizen's share of the national budget? Try to make an estimate first without a calculator or computer. Compare your two answers.

2. Using the figures above, what is each citizen's share of the national debt? Try to make an estimate first without a calculator or computer. Compare your two answers.

   Discussion: What exactly is the national debt anyway? The U.S. Treasury Department's web site may help you understand this:

www.treas.gov/education/faq/markets/national-debt.html
Basically, when the Congress decides to spend more than it makes in revenue it must borrow the extra amount - just like a person would. The government borrows money from people by promising to pay them back plus interest. They do this by selling to the public various forms of "IOUs," examples of which are Treasury Bills, Notes, United States Savings Bonds, etc. The national debt and budget are important because they can have an impact on our lives - such as what interest rates we pay and how much our taxes may be.

3. A TV program reported that a billion dollars in waste was discovered in government spending.

    a. What percent of the national budget is a billion dollars?
    b. If you had $100 and lost 0.05% (or 5/100 of 1%) of it, how much money did you lose?

4. Mr. Smith and Ms. Jones each run government departments. When the government auditors did their annual accounting they found that five hundred dollars was missing from Mr. Smith's department and that a hundred thousand dollars was missing from Ms. Jones' department. The accountants criticized Mr. Smith but gave Ms. Jones a good rating. Can you guess why? (See the next question.)

5. During an inquiry, the auditors revealed that Ms. Jones' department handled a billion dollars last year while Mr. Smith's department handled a hundred thousand dollars. (See question above.)

    a. What percent of Ms Jones's budget was missing?
    b. What percent of Mr. Smith's budget was missing?
    c. Can you explain the accountants' ratings now?

6. According to www.usaid.gov/pubs/cbj2003/request.html the total request for USAID (commonly called "foreign aid") is $8,477,724,000. What percent of the national budget is foreign aid?

The following questions are intended as a fun way to help us get an idea of how big a billion is. You might have fun guessing the answers before you compute them.

Suppose you have one billion dollars in one-dollar bills (a nice thought). Let's see just how big a stash you have.

7. Measure a one-dollar bill and record its length and width as accurately as you can.

8. If you laid a billion dollar bills end to end how many miles long would the trail be? About how many times around the earth would this string of bills reach? (The circumference of the earth is approximately 25,000 miles.)

9. If you laid them together on the ground in a big rectangle, how many square miles would they cover?

10. For the next three questions you need an estimate of the thickness of a dollar bill. How would you obtain that?

11. If you stacked them tightly how many miles high would the stack be?

12. How many cubic feet would they occupy?

13. Measure the dimensions of a room, such as your classroom. How many such rooms would a billion dollar bills fill?

14. You're filthy rich now, so you can afford to hire one of your friends to count your money for you. (I'm sure you can trust someone.) Let's say it takes about 5 seconds to count to ten out loud - that's about right, try it. (It may take longer when you get into the millions, but let's ignore that for now.) How long will it take your pal to count your money for you, assuming it's all there?

Instructor's Sheet

In addition to the example of money, one might consider this scenario: The distance from the Sun to the nearest star, Proxima Centauri, is 4.22 light years, which is roughly 25.4 trillion miles. *http://www.anzwers.org/free/universe/stardist.html*

If you had a rocket ship that went a million miles an hour, how long would it take to get to Proxima Centauri? (In orbit, a space shuttle circles the earth at a speed of about 17,500 mph, so our hypothetical space ship is going a lot faster.)

Why is an astronomical measurement like the one above important? For one thing it shows how unlikely it is that we will ever visit other stars. It also gives us an indication about how unlikely it is that extraterrestrial beings from other solar systems could (or would) come here - at least by anything like conventional means.

Answers to the problems are approximate and may vary depending on how accurate the estimated measurements are.

1. To estimate each citizen's share of the national budget note that the budget is about 2×10^{12} and the population of the United States is about 300×10^6 or 3×10^8 . From this we can estimate $2/3 \times 10^4$ or about $0.67 \times 10^4 = \$6700$. The calculated value is about $7000. (You might ask students to contemplate why their estimates were different from their calculated answers, if they were.)

2. To estimate each citizen's share of the national debt note that the debt is about 6×10^{12} and the population of the United States is about 300×10^6 or 3×10^8 . From this we can estimate 2×10^4 or about $20,000. The calculated value from the given figures is about $21,043. (You might ask students to contemplate why their estimates were different from their calculated answers, if they were.)

3. A TV program reported that a billion dollars in waste was discovered in government spending.

   a. What percent of the national budget is a billion dollars? (0.0487% or about 5/100 of 1%)
   b. If you had $100 and lost 0.05% you lost 5 cents.

4. *As the next question illustrates, if Jones' budget is much larger than Smith's this makes sense.*

5. During an inquiry, the auditors revealed that Ms. Jones' department handled a billion dollars last year while Mr. Smith's department handled a hundred thousand dollars.

   a. What percent of Ms Jones's budget was missing? (0.01% or 1/100 of 1 %)
   b. What percent of Mr. Smith's budget was missing? (0.005% or 1/2 of 1%)
   c. Can you explain the accountants' ratings now? *As a percentage of their budgets, Jones did much better than Smith.*

6. According to www.usaid.gov/pubs/cbj2003/request.html the total request for USAID (commonly called "foreign aid") is $8,477,724,000. That's 0.413% of the national budget. *Remark:* $8,477,724,000 is the FY 2003 Presidential request for appropriations under the Foreign Operations Subcommittee.
7. I got 6 3/16 inches long by 2 5/8 inches wide. Answers may vary depending on how accurate the estimated measurements are.

8. End to end they stretch 97,656 miles. That's nearly four times around the world.

9. They cover about 4 square miles.

10. Assume that a one-dollar bill is the same thickness as a page in a textbook. I measured the thickness of my book (minus covers) and got 1 1/8 inches. I divided this by the number of sheets of paper. My book had 842 pages, so I divided by 2 to get the number of sheets of paper: 421. Thus I concluded that a page in my textbook was approximately 0.00267 inches thick. I'll use 0.003 inches in the following calculations.

11. The stack reaches over 47 miles high. (47.35)

12. They occupy 28,198 cubic feet.

13. This depends on your room, of course. Let's take a classroom measuring 25 by 25 by 8 feet, so that's 5,000 cubic feet. Our money would fill over 5 1/2 such rooms. (5.6)

14. Counting to a billion continuously will take about 5,787 days. That's almost 16 years (15.85) not counting extra days for leap years.

Resources.

- *www.whitehouse.gov/omb/budget/fy2003/bud34.html*
- *www.brillig.com/debt_clock/*

- www.treas.gov/education/faq/markets/national-debt.html
- www.usaid.gov/pubs/cbj2003/request.html