Leonardo da Vinci's Tree and the Law of Channel Widths – Combining Quantitative Geomorphology and Art in Education

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ABSTRACT

About 1500, Leonardo da Vinci sketched a tree with arcs through each yearly growth of branches. In his mirror-image handwriting, and including an equation, he noted that the sum of the thicknesses of all the new branches produced each year would equal the sum of the thicknesses of branches from each previous year, down to and including the trunk. He wrote further that the same relation exists between a main watercourse and its branches. Leonardo's tree and notes clearly illustrate the principle of stream ordering in drainage network composition. Indeed, Leonardo combined network morphometry and hydraulic geometry, which together have thus far been inadequately investigated. Leonardo's tree drawing marks the discovery of quantitative drainage-network analysis, which was rediscovered by R.E. Horton more than four hundred years later. We use it to combine science and art in K-12 education.

Keywords: Education – precocce; history of geology; history of science; hydrogeology and hydrology; surficial geology – geomorphology.

Introduction

Leonardo da Vinci is arguably the most famous artist (The Mona Lisa; The Last Supper) of all time. He also is known as a musician, mathematician, inventor, engineer, architect, anatomist, astronomer, botanist, and writer (see Bramly, 1992, Reti, 1988, and Kemp and Roberts, 1989, for summaries and references, and MacCurdy, 1955, and Richter, 1883 and 1970, for comprehensive notebooks). Leonardo also was very interested in water (Mather and Mason, 1939; Krynine, 1960; DuBar, 1960; Rouse and Ince, 1963; Biswas, 1970; Meinhold, 1981; Alexander, 1982). He studied hydraulics, hydrology, and meteorology, filled volumes with notes and drawings depicting patterns of flow, channels, turbulence and clouds, and used these patterns in drawings and paintings of hair, streams, and landscapes. The Mona Lisa, probably the best known painting in the world, has a prominent river meander in the near background (Pestrong, 1994, p. 250).

One day, while perusing art books in the National Art Gallery bookstore, I (RGS) came across a drawing of a tree in a chapter on botany in a book on Leonardo (Figure 1, published in Kemp and Roberts, 1989, p. 216; Richter, 1970, plate 70; da Vinci, 1939, p. 375; Emboden, 1978, p. 111; and in Ravaisson-Mollien, 1890). When I saw the arcs Leonardo drew through the branches and then read his notes, I immediately recognized that he had scooped, by more than 400 years, the famous hydrologist Robert E. Horton, who is recognized as the discoverer (Horton, 1932; 1945) of the “laws of drainage composition” (Strahler, 1957; Schum, 1956). That is, by drawing arcs through those branches representing each year’s growth, Leonardo established a hierarchy of branches in the network, exactly the same as Horton’s method of network ordering as revised by Strahler (Figure 2).

Network Composition

Horton designated the smallest, unbranched tributaries (Figure 2) in a network as first-order channels. Two first-order channels come together to form a second-order channel, with the same numbering scheme continued on through the network to any point on a main stream. Using this scheme, and maps with a scale of an inch to a mile, for example, Leopold, Wolman, and Miller (1964, p. 142) found that the Mississippi River is a tenth order stream at its mouth. Horton further found that if the number, length, and drainage area of channels of each order are plotted against order on semilog paper, straight lines almost invariably result. From this fact he concluded that stream networks are organized consistently geometrically and are quite predictable mathematically. Today, drainage-composition analysis is taught as standard material in introductory geomorphology courses.

Leonardo’s Tree

When da Vinci drew arcs through the tree branches representing each year’s growth, he established a hierarchy of branches in the network. This was exactly the same as Horton’s method of stream-network ordering as later revised by Strahler (Figure 2).

Next to his drawing and in his right-to-left, mirror-image script, Leonardo noted that (translation in Richter, 1970, p. 205):

Every year when the boughs of a plant (or tree) have made an end of maturing their growth, they will have made, when put together, a thickness equal to that of the main stem; and at every stage of its ramification you will find the thickness of the said main stem; as: i, k, g, h, e, f, c, d, a, b, will always be equal to each other; unless the tree is pollard – if so the rule does not hold good.

All the branches have a direction which tends to the centre of the tree m.
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Figure 1. Leonardo da Vinci's tree illustrating his notes on the hierarchy of branching networks and his labeling of arcs (orders) as used in his equation. (From da Vinci, 1593.)

In another notebook on the same subject, Leonardo wrote (Richter, 1970, p. 205):

All the branches of a tree at every stage of its height when put together are equal in thickness to the trunk (below them).

All the branches of a water (course) at every stage of its course, if they are of equal rapidity, are equal to the body of the main stream.

Significance
The last statement above indicates that Leonardo understood the composition of streams in a drainage network. In addition, by describing equal thicknesses of tree branches, "rapidity" (velocity or discharge?) of watercourse "branches" (tributaries), and "body" (channel width or discharge?) of the main stream, Leonardo was clearly introducing the concept of downstream hydraulic geometry, which was rediscovered and presented over forty years ago by Leopold and Maddock (1953). Indeed, the literature contains few examples in which the relation of channel width and order has been adequately investigated (one such is Woldenberg, 1972), but its importance is becoming increasingly recognized today in geomorphology, for example in geographic-information-systems interfaces with hydrologic models (Shepherd, 1996). Of course one reason that hydraulic geometry analyses have only rarely included channel width is that measurements of width in the field, or from large-scale maps, are necessary to properly quantify the value. However, studies of other networks, such as those of the lungs or heart (see Jarvis and Woldenberg, 1984 for references) have, in fact, included width.

A different but equally important aspect of Leonardo's tree is how it can be used for combining quantitative geomorphology and art in education. First, a student is provided with a standard USGS seven-and-a-half minute quadrangle map, and the principles of maps and their symbols, contour lines, topography, stream networks, and watersheds are explained. Next, the student orders the stream network, obtains the numbers, lengths, and gradients of each stream segment of each order, and makes a semilog plot of order against the three variables, with Leonardo's tree as an example. This generally produces straight lines which can be described with exponential equations, which are discussed. Why this is so, and the fact that many other tributary networks follow these same "laws" are considered. Finally, using the data and relationships developed from the drainage network, the student uses a malleable grade of wire to create a "da Vinci tree" (Figure 3).

Figure 2. Horton's classification of stream orders, as applied to da Vinci's tree, and the network composition graph plotted from measurements of an enlarged copy of the original drawing.
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Figure 3. Leonardo da Vinci tree made from wire, with same-number composition as Comanche Creek watershed, central Texas.

Summary

Leonardo da Vinci reported and illustrated the principle of network composition more than 400 years before Robert E. Horton. Leonardo sketched a tree that follows compositional “laws” and stated that the principle applies to natural watercourses. Conversely, the composition of a natural drainage network can be used by students to create a da Vinci tree from wire, combining art and science in K-12 education.

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