Measuring the Earth Using a GPS

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Modified from:
Kerski, J.J., 2007, Measuring the Earth's Circumference with GPS, Copyright ESRI, jkerski@esri.com, www.esri.com/k-12.
Johnson, A. (ESRI) and Sullivan, D., (Monterey Peninsula College), 2006, Measuring the Earth---The Advantage of GPS Technology (adapted from an exercise created by Houston Community College under an NSF Grant for GIS Workplace Exercises).

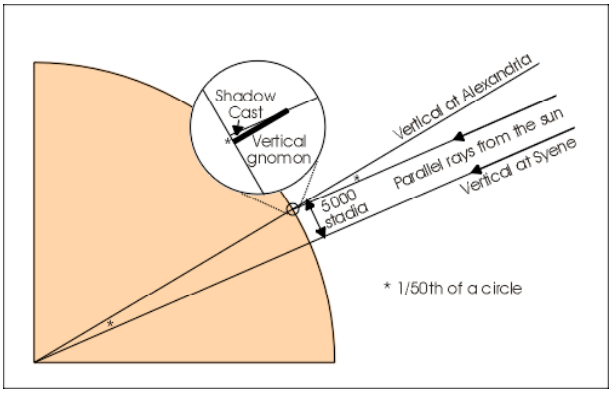
In this activity, students measure the circumference, radius, mass, and volume of the Earth using Global Positioning System (GPS) receivers.

Measuring the Earth's Circumference

Background

Eratosthenes (276 BC-194 BC, see picture right from http://www.nndb.com/people/712/000095427/eratosthenes-1-sized.jpg) was born in Cyrene, now part of Libya, in northern Africa. After studying in Alexandria and Athens, he became the director of the Great Library in Alexandria. The Library truly lived up to its name, housing a great deal of the learned and compiled knowledge of the time. It was at the library where Eratosthenes read about a deep vertical well near Syene (now Aswan) in southern Egypt. Once a year at noon at this well, on the day of the Summer Solstice, the bottom of the well was entirely lit by the sun. The sun was directly overhead, its rays shining straight into the well.

Eratosthenes then placed a vertical post at Alexandria, which was almost due north of Syene, and measured the angle of its shadow on the same date and time. Making the assumptions that (a) the earth is round and that (b) the sun's rays are essentially parallel, Eratosthenes knew from geometry that the size of the measured angle (7°12') equaled the size of the angle at the earth's center between Syene and Alexandria. Knowing also that the arc of an angle this size was ~1/50 of a circle, he then had to determine the distance between Syene and Alexandria. This was a difficult task during that time, due to different strides of camels and human error, and despite the best efforts of the King's surveyors, required years of effort. It was finally determined to be 5000 stadia. Eratosthenes multiplied 5000 by 50 to find the Earth's circumference (see figure below from http://www.fes.uwaterloo.ca/crs/geog165/images/eratosthenes.gif). His result, 250,000 stadia (about 46,250 km), was amazingly close to the accepted modern measurements (40,075 km around the equator and 40,008 km around the poles).



With your GPS, you too can emulate Eratosthenes' methods and measure the circumference of the Earth! By so doing, you are integrating geography, mathematics, Earth Science, and physics to solve an applied problem.

Measuring the Earth's Polar Circumference

1. Students should organize into teams of two sharing one GPS unit.

2. In each team, you will mark waypoints using the UTM (Universal Transverse Mercator), coordinate system, where the units are in meters. The numbers for UTM represent eastings (relative to the Central Meridian in the UTM zone, and northings (relative to the Equator). *Important: Set each unit to the datum **WGS 84** so that all students are working with the same datum (model of the Earth's shape).* To obtain decimal degree values (latitude-longitude coordinates) for each point, you can set the units to decimal degrees once you are back in the lab and access the waypoint information page for that waypoint to obtain the coordinates in decimal degrees. Alternatively, you could simply change the GPS units in the field and gather the waypoints in decimal degrees (or teams could use two GPS's, one in each coordinate system).

3. Locate a safe place away from buildings, cliffs, trees, power lines, and other obstacles that allows you to walk in a north/south direction for at least 200 meters - the longer the better. You can use the GPS coordinates to determine a true north-south track.

4. Take a GPS reading at the north end of your chosen track (Note: If you can "average" the points and then mark the waypoint, please do so (and make sure that WAAS correction is enabled). Name the waypoint a meaningful name so that you can identify it later. Record the UTM coordinates in Table 1. You can record the latitude and longitude from this stored waypoint later in the lab or you can switch units in the field and gather a new waypoint in decimal degrees (see step 2 above). Enter the value for decimal degrees in Table 1.

5. Repeat for the south end of your chosen track.

NOTE: Do not round off! Please retain a minimum of 5 places to the right of the decimal point.

Table 1. Measuring the Earth's Polar Circumference

North Point (Latitude; *decimal degrees*)	a.
North Point (Longitude; *decimal degrees*)	b.
North Point (UTM northing; *meters*)	c.
North Point (UTM easting; *meters*)	d.
South Point (Latitude; *decimal degrees*)	e.
South Point (Longitude; *decimal degrees*)	f.
South Point (UTM northing; *meters*)	g.
South Point (UTM easting; *meters*)	h.
Distance in decimal degrees [**a**-**e**°]	i.
Distance in meters [**c**-**g** *meters*]	j.
Knowing the ratio: $\frac{\text{Distance in decimal degrees [i]}}{360°} = \frac{\text{Distance in meters [j]}, \text{[PC] } meters}{}$ we can solve to find the Earth's polar circumference: $\text{[PC] } km = \frac{(\text{[j] } meters * 360°)}{\text{[i]}°} * \frac{1}{1000}$	k.
To calculate % error (assuming a true polar circumference of 40,008 *km*): $\text{[PC] \% error} = \frac{\lvert \text{[k] } km - 40{,}008\ km \rvert}{40{,}008\ km} * 100$	l.

Measuring the Earth's Equatorial Circumference

1. Follow steps 1 & 2 for **Measuring the Earth's Polar Circumference** above.

2. Locate a safe place away from buildings, cliffs, trees, power lines, and other obstacles that allows you to walk in an east/west direction for at least 200 meters - the longer the better *(Note: you may want to use one of the points used for the polar circumference calculation)*. You can use the GPS coordinates to determine a true east-west track.

3. Take a GPS reading at the east end of your chosen track (Note: If you can "average" the points and then mark the waypoint, please do so (and make sure that WAAS correction is enabled)). Name the waypoint a meaningful name so that you can identify it later. Record the UTM coordinates in Table 2. You can record the latitude and longitude from this stored waypoint later in the lab or you can switch units in the field and gather a new waypoint in decimal degrees (see step 2 above). Enter the value for decimal degrees in Table 1.

4. Repeat for the west end of your chosen track.

NOTE: Do not round off! Please retain a minimum of 5 places to the right of the decimal point.

Table 2. Measuring the Earth's Equatorial Circumference

West Point (Latitude; *decimal degrees*)	a.
West Point (Longitude; *decimal degrees*)	b.
West Point (UTM northing; *meters*)	c.
West Point (UTM easting; *meters*)	d.
East Point (Latitude; *decimal degrees*)	e.
East Point (Longitude; *decimal degrees*)	f.
East Point (UTM northing; *meters*)	g.
East Point (UTM easting; *meters*)	h.
Distance in decimal degrees [**b**-**f** °]	i.
Distance in meters [**h**-**d** *meters*]	j.
Knowing the ratio: $\frac{\text{Distance in decimal degrees [i]}}{360°} = \frac{\text{Distance in meters [j], [LC] } meters}{}$ we can solve for the circumference of the Earth along this line of latitude [**LC**] in *kilometers*: $[\mathbf{LC}]\ km = \frac{([\mathbf{j}]\ meters * 360°)}{[\mathbf{i}]°} * \frac{1}{1000}$ and to determine the equatorial circumference [**EC**] in *kilometers (use degrees not radians)*: $[\mathbf{EC}]\ km = \frac{[\mathbf{LC}]}{\cos(\text{latitude } [\mathbf{a}])}$	k.
To calculate % error (assuming a true equatorial circumference of 40,075 *km*): $[\mathbf{EC}]\ \%\ \text{error} = \frac{\lvert\ [\mathbf{k}]\ km - 40{,}075\ km\ \rvert}{40{,}075\ km} * 100$	l.

Polar Circumference & Equatorial Circumference

1. Discuss how your calculated polar circumference compares with both the accepted polar circumference and with Eratosthenes' measured circumference. Give multiple reasons for why your answer is not exactly the same as the accepted circumference of the Earth.

2. What do your polar and equatorial circumference calculations tell you about the shape of Earth?

3. Using your calculated values, how long would it take to walk around the Earth along the polar circumference (in *days*)? Along the equatorial circumference (in *days*)? Along the circumference of the line of latitude in Table 2 (in *days*)? To determine this, set the receiver to the page where you can determine how fast you are moving (*km/hr*). Show your work.

   Polar circumference _____ *days*

   Equatorial circumference _____ *days*

   Latitudinal (_____°) circumference _____ *days (use the latitude from Table 2**a**)*

Determining the Mass and Volume of the Earth

Determine the length of the polar circumference (**Table 1k**).	a.
Knowing that the circumference of a circle is: $2 * \pi * R$ *where R=radius of the Earth.* $[\mathbf{R_{km}}]\ km = \frac{[\mathbf{a}]\ km}{(2 * \pi)}$ $[\mathbf{R_m}]\ meters = [\mathbf{R_{km}}] * 1000$	b.
Mass Knowing that mass is: $\text{Mass} = \frac{(\text{acceleration} * R^2)}{G}$ *where acceleration due to gravity = 9.8 meters/s^2, and G, the constant of proportionality, is 6.67 x10^{-11} m^3/ kg $second^2$.* You can determine the mass (*kg*) of the earth using the Earth's radius [$\mathbf{R_m}$] from [**b**].	c.
To calculate the mass % error (assuming a true mass of 5.98x10^{24} *kg*): $\%\ \text{error} = \frac{\lvert [\mathbf{c}]\ kg - 5.98\text{x}10^{24}\ kg \rvert}{5.98\text{x}10^{24}\ kg} * 100$	d.
Volume Knowing that volume is: $\text{Volume} = 4/3 * \pi * R^3$ You can determine the volume (km^3) of the earth using the Earth's radius [$\mathbf{R_{km}}$] from [**b**].	e.
To calculate the volume % error (assuming a true volume of 1.0975095x10^{12} km^3): $\%\ \text{error} = \frac{\lvert [\mathbf{e}]\ km^3 - 1.0975095\text{x}10^{12}\ km^3 \rvert}{1.0975095\text{x}10^{12}\ km^3} * 100$	f.