Estimation in physical sedimentology

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Intro comments

(1) Most undergrad sed-strat books have a chapter or two on fluid mechanics. Many are not very good, and in any case the focus in sed geo should be on sediment dynamics, not fluid dynamics.

(2) Sediment dynamics begins with Exner equation (mass balance — more on this in Exner talk)

in words: rate of dep = (flux in - flux out) + fallout from suspension

the most important term → focus on sediment flux!
(we'll also learn about the flow, since flow → sed flux)

(3) goal of quantitative training is (we hope!) "insight not numbers" but numbers → insight

- before students can learn to use calculus, numerical models, etc. effectively they must get in the habit of thinking quantitatively

- emphasize science, not math

Intro / setup remarks

(1) The focus in sed geo should be on <u>sediment</u> dynamics, not <u>fluid</u> dynamics.
→ Fluids are secondary!

(2) Sediment dynamics <u>begins</u> <u>with</u> <u>Exner</u>

(w/out creation or destruction)

(more on this in Exner talk)

mass gain / loss = flux in − flux out

↗ we need to estimate this!

Key parameters for sediment flux:

(3) The goal of quantitative training is "insight not numbers"

- before students can learn to use calculus, numerical models, etc. effectively they must get in the habit of <u>thinking</u> <u>quantitatively</u>

- simple estimation aims at <u>insight</u> not numbers! Emphasize science, not math

Sediment flux parameters

dimensional: q_s : $\frac{\text{Volume sed}}{\text{time} \cdot \text{width}}$ ← or mass

therefore: $\frac{L^2}{T}$

q_s is controlled by:

(1) applied force: shear stress τ_o

two simple forms for estimation:

(1.1) $\tau_o = \rho g h S'$ gravity sliding on slope S' balances friction τ_o

h: depth or thickness

NOT JUST FOR FLUIDS!

(1.2) $\tau_o = c_f \rho u^2$ faster flow ⇒ more friction

c_f is a drag coefficient; typical value ~ 0.01

[check dimensions for yourself!]

We also need:

(1.3) nondimensional stress for sediment transport

(2) Grain properties; resistance & transport properties

(2.1) grain size - influences weight but also (less obvious but more important) relation of surface to body forces

stickiness, cohesion ← surface; body forces → gravity

estimation exercise:

estimate ratio of surface forces to body forces

surface force must increase w/ sfc area

problem: don't know formula for surface area of a sphere!

Do not look it up - think instead:

by dimensional reasoning: $A \sim D^2$

body force (gravity) must go as $V \sim D^3$

so: sfc force/body force $\sim D^{-1}$

coarse sediment (usually) non-cohesive and v/v

Putting these together we get something useful:

(3) nondimensional stress for sediment transport (non cohesive → weight limited)

also called "Shields stress" after Albert F. Shields

$$\tau_* = \frac{\tau_0}{\rho g (s-1) D}$$

CHECK (NON)DIMENSION!

grain diameter

sed specific gravity

derivation: applied tractive force $\sim \tau_0 D^2$

immersed weight $\sim g(s-1)\rho D^3$

Note: $\frac{\tau_0}{\rho}$ has units $\frac{L^2}{T^2}$ so $\left(\frac{\tau_0}{\rho}\right)^{1/2}$ is $\frac{L}{T}$

(velocity!) - called "shear velocity" u_*

handy measure of turbulence intensity:

rms ("typical") turbulence velocity $\sim u_*$

also: $g(s-1)D$ has units $\frac{L^2}{T^2}$ too

To finish, return to sediment flux:

(4) We can non-dimensionalize sed flux using the same parameters:

$$q_{s*} = \frac{q_s}{\underbrace{[g(s-1)D]^{1/2}}_{L/T} \cdot \underbrace{D}_{L}} = \frac{q_s}{\underbrace{[g(s-1)]^{1/2}}_{\text{MKS} \cong 4} D^{3/2}}$$

called the "Einstein" nondimensional sed flux

↳ A. Einstein's son Hans

sediment flux is calculated empirically even for uniform, spherical, constant-density grains

There are dozens of formulas but if you only know one, know this one:

$$q_{s*} = \frac{0.05}{c_f} \tau_*^{5/2} \quad \text{Engelund \& Hansen}$$

With this background & some empirical knowledge, we can estimate a lot from a little in the field!

Some basic information:

(1) critical τ_* for getting grains in motion:

$\ll 1$: easier to roll a ball than pick it up

range ~ 0.02 – 0.05 for silt size & up

(2) for gravel-bed rivers, transport generally occurs for τ_* not far above critical, i.e.

Note: these low values keep gravel rivers in lower plane bed regime, mostly

$0.05 < \tau_* < 0.1$ A value around 0.07 is typical

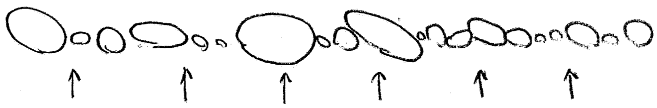
[why? bank erosion → widening → reduction in τ_* if τ_* gets too uppity]

(3) for sand-bed rivers, generally $1 < \tau_* < 2$

[why? not known!]

Estimation - gravel river deposits

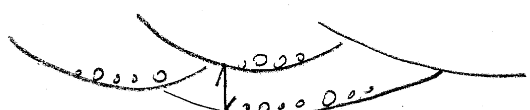
(1) estimate median grain size (D_{50})



stretch a tape along a bed & measure b-axis length on each grain chosen at a fixed interval
→ you are trying to simulate Wolman sampling

→ 50 clasts is enough to get the median
Teaching note: D_{50} will be less than you thought!

(2) estimate paleo depth

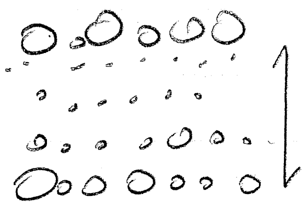


cross cutting channels

average preserved depth ~ 70% average original depth

max. channel depth

lateral accretion w/ preserved topo: may overestimate mean depth by ~ 2



FU sequences - if that's all you have ...

Now you can estimate paleoslope:

$$\tau_* = 0.07 = \frac{\cancel{\rho g} h S}{\cancel{\rho g}(s-1) D_{50}}$$ students should know $s-1 \cong 1.65$

$$S = \underbrace{0.07\,(s-1)}_{\text{about } 0.1} \left(\frac{D_{50}}{h}\right)$$

<u>Example:</u>

$D_{50} = 0.02$ m $\quad h = 1.5$ m

$S \simeq 1.3 \times 10^{-3}$ [quite reasonable]

uncertainty is about a factor 2 → one digit is enough

Teaching note: a bed with $D_{50} = 2$ cm would look pretty coarse, i.e. would have plenty of grains with $D \sim 5$ cm and up

Inquiring minds want to know: how far can this be transported ??

<u>Answer</u>: $S \sim 10^{-3}$ means 1 km of uplift (or accumulated sediment) for 1000 km of distance. Is that <u>too</u> <u>much</u> to ask for ?

Estimation — sandy river deposits

(1) Estimate grain size & paleodepth: same methods as above <u>except</u>

→ you (of course) use a hand lens and/or comparator to estimate D_{50}

→ if you have dune cross-strata, you can cross-check the depth, or estimate it without preserved channel forms:
mean set thickness ≃ 0.7 mean dune height
flow depth ≃ 5 * dune height
[i.e. depth ≃ 7 * set thickness]

Using $\tau_* = \dfrac{h S}{(s-1) D} \simeq 1-2$ for sand-bed rivers

a reasonable estimate is: $S \simeq \dfrac{2D}{h}$

Example: $D = 200\ \mu m = 0.2\ mm$
$h = 2\ m$

$S = 2 \times 10^{-4}$ [very reasonable sand-bed river value]

requires: 200 m ramp height over 1000 km

Further estimation (gravel or sand)
(but we'll use the sand example)
We can of course estimate τ_0 :

$$\tau_0 = \rho g h S' \simeq 4 \frac{kg}{m-s^2} = 4 \frac{N}{m^2} = 4\ Pa$$

hard for students to relate to
but equals hydrostatic pressure
from 0.4 mm (yes, mm) of water. Is that all??

Easier to relate to is the flow speed:

$$\frac{\tau_0}{\rho} = c_f u^2$$ (remember c_f = drag coeff. ~ 0.01)

$$u \simeq [100\, g h S']^{1/2} \simeq 0.6\ m/s$$ [ok?]
60 cm/s

if you like Froude numbers you can do that too:

$$\mathbb{F}r = \left[\frac{u^2}{gh}\right]^{1/2} = \left[\frac{S'}{c_f}\right]^{1/2} \simeq 10\sqrt{S'} \simeq 0.14$$

subcritical ... but you already knew that ...

With a little more work we can estimate the relative degree of suspension, which we can compare directly with the deposit itself:

suspension transport means grains are supported by interaction w/ vertical component of turbulence
$\longrightarrow$ physical criterion:

$$W'_{rms} \simeq W_s$$

particle settling velocity

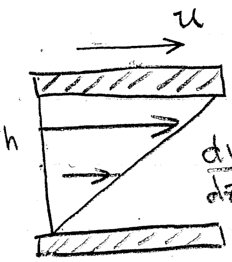
"typical" vertical turbulence velocity

and $W'_{rms} \simeq \left[\frac{\tau_o}{\rho}\right]^{1/2} \equiv u_*$

u_* is easy: $u_* = [g h \cdot S]^{1/2} \simeq 0.06$ m/s $= 6$ cm/s

$(u_* \simeq 0.1\, \bar{u})$

W_s is a little harder. We'll need the viscosity of water, which can be expressed in two forms:



dynamic: $\mu = \frac{\tau}{\left(\frac{du}{dz}\right)}$ $\quad \frac{du}{dz} = \frac{u}{h}$

the one you read about

Kinematic $\nu = \frac{\mu}{\rho}$

the one you use, mostly

For small w_s and D settling is viscous — use

<u>Stokes' Law</u>: $w_s = \frac{g(s-1)D^2}{18\nu}$ $\quad w_s D/\nu \lesssim 1$

ν for water $= 10^{-6}\ \frac{m^2}{s}$

for quartz on Earth in water:

$w_s \cong 10^6 D^2$ which works for $D \lesssim 100\ \mu m$
[MKS units]

examples:

$D = 1\ \mu m = 10^{-6}\ m \longrightarrow w_s \cong 10^{-6}\ m/s$
(clay size) falls ~ 1 diameter in 1 second

$D = 10\ \mu m = 10^{-5}\ m \longrightarrow w_s \cong 10^{-4}\ m/s$
falls ~ 10 diameters in 1 second

$D = 100\ \mu m = 10^{-4}\ m \longrightarrow w_s \cong 10^{-2}\ m/s = 1\ cm/s$
falls ~ 100 diameters in 1 second

For $100\ \mu m < D < 1\ mm$ the settling velocity depends on both viscosity and pressure — awkward!

$D = 0.25\ mm \quad w_s \cong 3\ cm/s$
$D = 0.5\ mm \quad w_s \cong 7\ cm/s$
$D = 1\ mm \quad w_s \cong 13\ cm/s$
range for med-coarse sand: 5-15 cm/s

[you can <u>roughly</u> use a linear increase from 0.1 - 1 mm]

For $D > 1\,mm$ settling is resisted by unequal "impact" pressure across the grain. You can use:

pressure force = immersed weight

$$\frac{1}{2}\rho C_D w_s^2 \left(\frac{\pi}{4}D^2\right) = g\rho(s-1)\left(\frac{\pi}{6}D^3\right)$$

$$w_s = \left[\frac{4}{3}\frac{(s-1)}{C_D}gD\right]^{1/2}$$

$C_D \simeq 1$ for typical grains

$$w_s \simeq 5\sqrt{D} \quad \text{in MKS}$$

e.g. $D = 0.01\,m \longrightarrow w_s \cong 0.5\,m/s$

$\longrightarrow$ but you will rarely have cause to use this because grains this large are rarely suspended

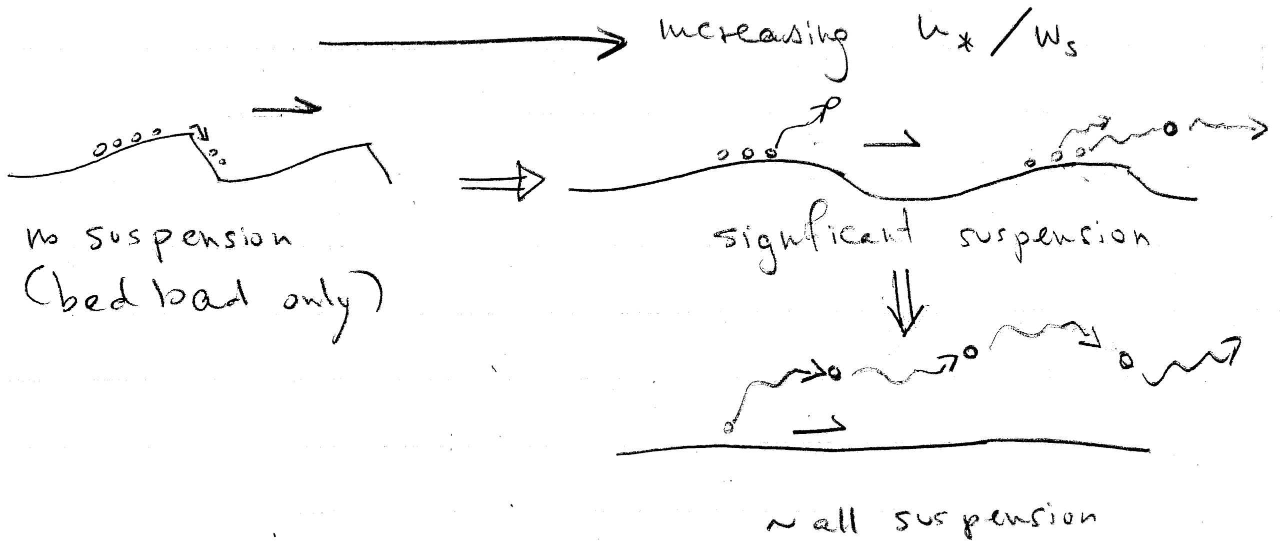
Getting back to our example:

for $D = 200\,\mu m = 0.2\,mm$ $\quad w_s \cong 2\,cm/s$

we had (p.11): $\quad u_* = 6\,cm/s$

These numbers are comparable $\rightarrow$ significant amounts of sediment in suspension ("mixed-load"

transport: bed load + suspension)
bed forms lengthen, lower, then wash out:



[if you want to go a little deeper, you can introduce the "Rouse Number"

$$Ro = \frac{w_s}{\kappa u_*} \qquad \kappa = 0.4 \text{ (Von Karman's constant)}$$

strong suspension if $Ro > 1$

With significant suspension, it also makes sense to estimate the advection distance (distance over which a grain can settle out of the flow)

$$L_a = \frac{hU}{w_s} = 60 \text{ m}$$

[approx. length scale for deposition due to deceleration]

We can also estimate sediment flux (note: we need only D for this ...)

for a typical $\tau_* \simeq 1$ and $c_f \simeq 0.01$

$q_{s*} \simeq 5$ (p. 5) and so $q_s = 5\left[g(s-1)D\right]^{1/2} D$

$D = 2\times10^{-4}$ m & $D^{1/2} \cong 1.4\times10^{-2}$ $m^{1/2}$

$\rightarrow D^{3/2} \simeq 3\times10^{-6}$ $m^{3/2}$

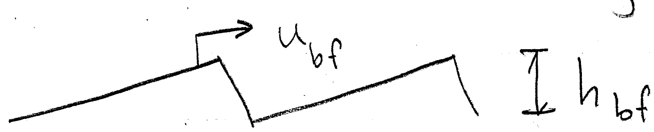
$\left[g(s-1)\right]^{1/2} \simeq 4$ $m^{1/2}/s$

so $q_s \sim 6\times10^{-5}$ m^2/s

In our sand-bed example there would be dunes present, so:

using bedform height $\cong 0.2h = 0.4$ m

then if we know how much sand is moving as bed load, we could estimate bedform migration speed as:



$q_{sbl} \cong \frac{1}{2} u_{bf} h_{bf} \rightarrow u_{bf} = 2q_{sbl}/h_{bf}$

bed load flux

Estimating that perhaps $\frac{1}{2}$ the flux is bed load (the rest suspension) we get:

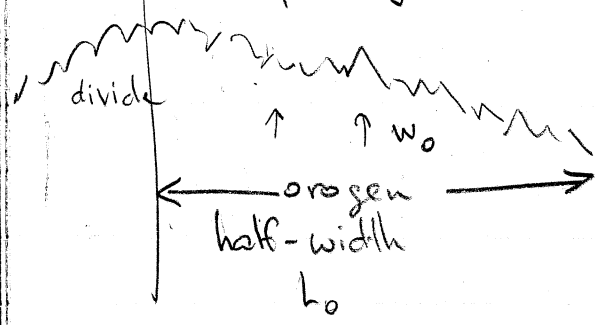
$$u_{bf} \cong 1.5 \times 10^{-4} \text{ m/s}$$, i.e. in an hour, they'd advance ~0.5 m

pretty slow — but a 10 m long bed, 0.4 m thick require only 20 hr to form

→ teaching note: this is where I would start talking about stratigraphic completeness — you have a pretty good idea how how long it took to form one bed... count beds in the outcrop — add up the time...

another way to get at the same thing:

Estimate sediment flux from source area (plausible order of magnitude)



if mechanical weathering ≃ chemical weathering then:

$$q_{so} \simeq L_o w_o / 2$$

get students to come up with plausible values for L_o & w_o

e.g. w_0 should be ~ 0.1 horizontal plate motion rates, ~ 1 mm/yr

so if $L_0 = 100$ km $\quad q_{s0} \sim \dfrac{(10^5 \text{ m})(10^{-3} \text{ m/yr})}{2}$

$q_{s0} \sim 50 \text{ m}^2/\text{yr}$

handy fact: $1 \text{ yr} \cong \pi \times 10^7 \text{ s} \simeq 3 \times 10^7 \text{ s}$

$q_{s0} \simeq 1.6 \times 10^{-6} \text{ m}^2/\text{s}$

if ~ 30% of this is sand, $q_{s0s} \simeq 5 \times 10^{-7} \text{ m}^2/\text{s}$

→ lower than our previous estimate of q_s but only by 2 orders of magnitude – easily accounted for by e.g. floods (~3 days of activity per year → intermittency factor of ~ 0.01)

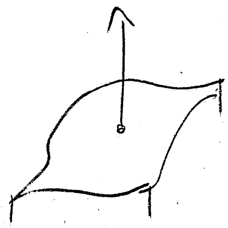
I hope you have enjoyed these notes and find them useful in your teaching.

→ No calculators were harmed to produce these notes!

Using Exner in undergrad sed/strat

→ This is more important & useful than nearly all of the fluid mechanics in undergrad sed/strat texts, but is absent or presented too late to be useful

Sediment mass balance
↳ we'll use volume



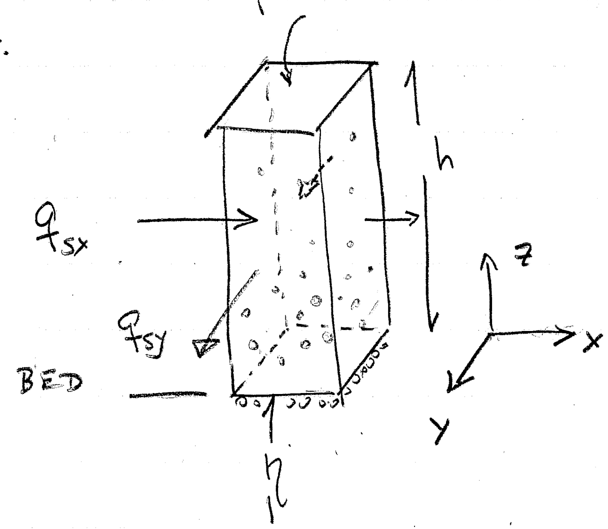
happens ONLY IF

deposition: ADD sed. volume to bed { flux in > flux out } AND/OR { fallout from flow }

no sed. flow at top

assuming no creation or destruction

in math:



q_s : $\frac{\text{vol sed}}{\text{width} \cdot t}$
"unit sediment flux" $L^2 T^{-1}$

Exner:

$$\underbrace{\overset{\text{subs. rate}}{\sigma} + \frac{\partial \eta}{\partial t}}_{\text{total rate of dep.}} = -\frac{1}{\underset{\text{bed sed. conc.}}{C_0}} \left[\underbrace{\frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y}}_{\text{flux in} - \text{flux out}} + \underbrace{\frac{\partial}{\partial t} \int_{\eta}^{\eta + h} \overset{\text{vol. sed. conc.}}{C(z)}\, dz}_{\text{gain/loss to susp.}} \right]$$

In more detail:

$\sigma + \dfrac{\partial \eta}{\partial t}$ rate of lengthening of sediment column

σ: basement $\downarrow$ or $\frac{\partial \eta}{\partial t}$: bed $\uparrow$

L/T

>0 RECORD

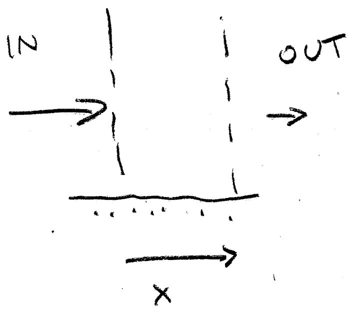
$=0$ PAUSE

<0 ERASE

SPATIAL

$-\dfrac{\partial q_{sx}}{\partial x}$

$L^2T^{-1}/L \rightarrow L/T$



IN > OUT $\rightarrow$ net deposition

[same for $\partial q_{sy}/\partial y$]

TEMPORAL

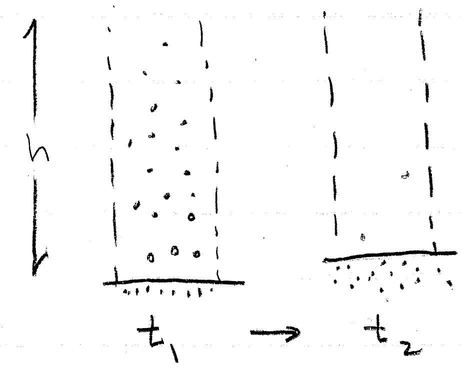
$$-\frac{\partial}{\partial t}\underbrace{\int_{\eta}^{\eta th} c(z)\,dz}_{L}$$

T^{-1}

c = vol. sediment / total flow vol. [nondim]

$\int c\,dz$ = vol. sed. / unit area

i.e. add up all sed. in column



sed. settled out $\rightarrow$ added to bed $\rightarrow \uparrow \eta$

Examples

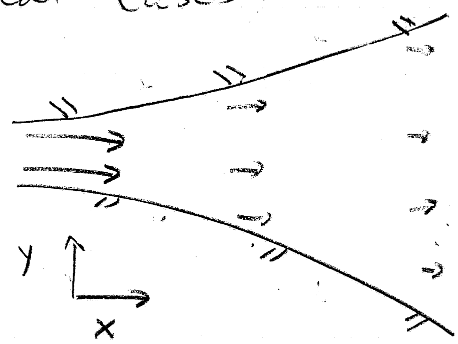
Spatial term: in general $q_{ts} \sim \tau^a$ $a>1$

bed shear stress

and $\tau \sim u^2$ ← flow speed

so $\frac{\partial q_{ts}}{\partial x} < 0$ generally means $u \downarrow$ in x

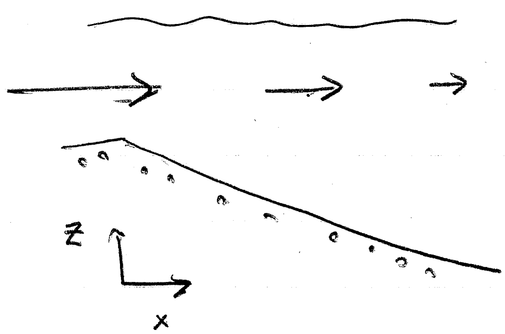
typical cases:



flow widens

deltaic or splay deposition!

or



flow deepens

foreset deposition

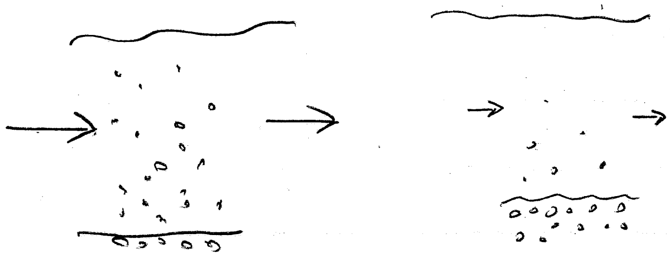
Temporal term:

in general $c \sim \tau^b$ $\quad b > 1$

so $\frac{\partial}{\partial t}\int c\,dz < 0$ generally means $u \downarrow$ in t

typical case:

__surge__ (flood, turbidity current, storm) in which flow speed _diminishes_ with time



__Note__: this term __must__ decrease in importance relative to spatial term on long (geologic) time scales

to see why, we scale the equation (estimate magnitude of each term)

[more compact notation]

$$\sigma + \frac{\partial \eta}{\partial t} = \frac{-1}{C_o}\left[\vec{\nabla}_H \cdot \vec{q}_s + \frac{\partial}{\partial t}\int_h c\,dz \right]$$

$$\sigma_r \qquad r \qquad [\sim 1] \qquad \frac{q_{sr}}{L} \qquad \frac{C_r}{T}$$

r denotes a "typical" or reference value

non-dimensionalizing using q_{sr}/L:

$$\frac{\sigma_r L}{q_{sr}} \qquad \frac{rL}{q_{sr}} \qquad 1 \qquad \frac{C_r L}{q_{sr} T}$$

short time scales

$$\frac{rL}{q_{sr}} \gg \frac{\sigma_r L}{q_{sr}}$$ by observation (Sadler ff.) (ok to drop σ)

$$\frac{C_r L}{q_{sr} T}$$ may be > 1 or not depending on rate of change in t vs. x or y (prev. page)

but as $T \to \infty$

$$\frac{C_r L}{q_{sr} T} \to 0$$

so: short-term Exner:

$$\frac{\partial \eta}{\partial t} = \frac{-1}{C_o}\left[\vec{\nabla}_H \cdot \vec{q}_s + \frac{\partial}{\partial t}\int_h c\,dz \right]$$

long-term:

$$\sigma + \frac{\partial \eta}{\partial t} = \frac{-1}{C_o}\,\vec{\nabla}_H \cdot \vec{q}_s$$

IMPORTANT NOTE:

neither of the basic terms causing deposition in the Exner equation depends on the value of q_s (or τ or U) at a specific place and time:

ONLY GRADIENTS MATTER!

Students often think "slow flow = deposition, fast flow = erosion". While there is some truth to this for cohesive (fine) sediment, for noncohesive particles it is wrong. Instead,

→ ["acceleration [space or time] → erosion
deceleration [space or time] → deposition"

This is why it is possible to deposit parallel laminated sand!